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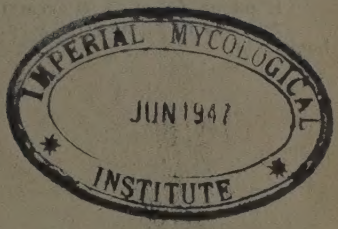

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TABLE DES MATIÈRES

MATHÉMATIQUES Pages

M. Olevsky. Solution du problème de Cauchy pour l'équation des ondes dans un espace à n dimensions à courbure constante 3

MÉCANIQUE

N. A. Slezkin. Penetration of a Thin Plate into a Viscous Medium 7

PHYSIQUE APPLIQUÉE

G. G. Polozova and **N. S. Sventitsky.** Band Heads of AlO Spectrum as an Inner Standard in the Spectrum Analysis of Aluminium Alloys 10

ÉLECTRICITÉ

P. A. Sviridenko. Stable Speeds of Rotation of an Asynchronous Machine with Single-Phase Stator and Rotor 13

CHIMIE PHYSIQUE

B. V. Nekrassov. Nuclear Distances and Bond Multiplicity 17

V. V. Tarassov. On a Theory of Low-Temperature Heat Capacity of Linear Macromolecules 20

GÉOLOGIE

A. G. Betekhtin. On Metamorphosed Manganese Deposits 24

MINÉRALOGIE

I. D. Sedletzky and **S. M. Yussupova.** Variation in Parameters of Montmorillonite as Affected by Petroleum 27

HISTOLOGIE

E. Kirpichnikova. Ultramicroscopic Structure of Spermatozoon 31

MICROBIOLOGIE

K. S. Sukhov. Formation of Necroses on Leaves of a Hybrid *Nicotiana tabacum*×*N. glutinosa*, Induced by the Virus of Tobacco Mosaic under Conditions of Plasmolysis 34

PHYSIOLOGIE VÉGÉTALE

A. A. Nichiporovich and **E. F. Ivanitzkaya.** On the Working Conditions of the Foliage as a Factor of Differentiation of Laticiferous Vessels in Koksaghyz Roots 36

A. A. Richter, Member of the Academy, **K. T. Sukhorukov** and **L. A. Ostapenko.** Photosynthesis and Development of Plants 40

PHYSIOLOGIE

A. L. Judeles, I. J. Postovsky and **T. I. Kazantzeva.** To the Question of Determining Antisulphanilamide Factors in Blood 42

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Moscou 1945

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MATHÉMATIQUES

SOLUTION DU PROBLÈME DE CAUCHY POUR L'ÉQUATION DES ONDES DANS UN ESPACE À n DIMENSIONS À COURBURE CONSTANTE

Par M. OLEVSKY

(Présenté par S. L. Soboleff, de l'Académie, le 13.III.1944)

1. Soit

$$ds^2 = \sum_{\alpha,\beta=1}^{n} g_{\alpha\beta}\, dx^\alpha\, dx \tag{1}$$

l'élément linéaire de l'espace $R_n (n > 2)$ à courbure constante k. Le problème de Cauchy pour l'équation des ondes dans R_n consiste à trouver une fonction u vérifiant l'équation

$$\Delta_2 u = \partial^2 u / \partial t^2 \tag{2}$$

et les conditions

$$u(P, t)\,|_{t=0} = f(P) \tag{3}$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = \varphi(P) \tag{4}$$

Ici f et φ sont des fonctions données, Δ_2 est le second paramètre différentiel de Beltrami par rapport à la forme (1). La solution de ce problème est l'objet de la présente Note.

2. Sans restreindre la généralité on peut se borner au cas où

$$f(P) = 0 \text{ *} \tag{3'}$$

En tenant compte de l'équation généralisée de Darboux (1), par les méthodes, développées en (2), on peut démontrer le théorème:

Si la fonction φ *est* $\frac{n+1}{2}$ *fois dérivable pour* n *impair* $\left(\text{et respectivement } \frac{n+2}{2} \text{ fois dérivable pour } n \text{ pair}\right)$, *la solution de* (2), (3'), (4) *peut être présentée sous la forme:*

$$u(P, t) = \frac{\sqrt{\pi}}{2^{\frac{n-1}{2}} \left(\frac{n}{2}\right)} \int_0^t J_0\left(\frac{n-1}{2}\sqrt{k(z^2 - t^2)}\right) \frac{d}{dz} L_{\frac{n-3}{2}} \psi(P, z)\, dz \tag{5}$$

$$\psi(P, z) \equiv \left(\frac{\sin\sqrt{k}\, z}{\sqrt{k}}\right)^{n-2} \overline{\varphi}(P, z)$$

* Si $u\{\varphi\}$ est la solution de (2), (3'), (4), la solution de (2), (3), (4) est de la forme $u\{\varphi\} + \frac{\partial}{\partial t} u\{f\}$.

où $L_{\frac{n-3}{2}}\Phi(z) \equiv \left(\frac{\sqrt{k}}{\sin\sqrt{k}z}\frac{d}{dz}\right)^{\frac{n-3}{2}}\Phi(z)$, *c'est-à-dire pour* $n=2m+1$ *elle est la* $m-1$*-ième itération de* $L \equiv \frac{\sqrt{k}}{\sin\sqrt{k}z}\frac{d}{dz}$ *effectuée sur* $\Phi(z)$, *et pour* $n=2m$ *on a* $L_{\frac{n-3}{2}} \equiv L_{m-1}L_{-1/2}$, *où* $L_{-1/2}\Phi(z) = \frac{1}{\sqrt{\pi}}\int_0^z \frac{\sin\sqrt{k}x\Phi(x)\,dx}{\sqrt{\cos\sqrt{k}x-\cos\sqrt{k}z}}$.

La fonction $\overline{\varphi}(P, z)$ *désigne la valeur moyenne de la fonction* φ *sur la sphère (au sens de la métrique* (1) *dans* R_n) *de rayon* z *et de centre* P. *Dans le cas où* $k>0$, *la fonction* $\overline{\varphi}(P, z)$ *doit être prolongée dans* $\pi/\sqrt{k} \leqslant z \leqslant 2\pi/\sqrt{k}$ *de manière à être impaire, et ailleurs de manière à être périodique (de période* $2\pi/\sqrt{k}$).

Pour $k \to 0$ (5) se transforme en formules classiques de Poisson — Volterra (3) et prend la forme commune pour le cas de n pair et impair (ce que l'on obtient au moyen de dérivation d'ordre fractionnaire).

En intégrant (5) par parties on trouve dans le premier terme du second membre une généralisation immédiate de la forme de la solution de Poisson — Volterra (pour $k \to 0$). Or, la présence du second terme fait voir que la diffusion des ondes dans R_n a lieu aussi pour $n=2m+1$ (4).

3. Montrons brièvement la méthode pour obtenir la solution (5)*. Au lieu de la fonction $u(P, t)$ nous cherchons d'abord $V(u], P, s, t)$ — la valeur moyenne de la fonction cherchée sur la sphère $F_s(P)$ (au sens de la métrique (1)) de centre P et de rayon s. En faisant usage de la relation (2') établie en (1), on arrive en vertu de (2) à l'équation suivante pour V:

$$\frac{\partial^2 V}{\partial s^2} + (n-1)\sqrt{k}\operatorname{ctg}\sqrt{k}s\frac{\partial V}{\partial s} = \frac{\partial^2 V}{\partial t^2} \tag{6}$$

En prenant les moyennes pour (3') et (4), nous obtenons les conditions initiales suivantes pour V:

$$V|_{t=0} = 0 \tag{7}$$

$$\left.\frac{\partial V}{\partial t}\right|_{t=0} = \overline{\varphi}(P, s) \tag{8}$$

où $\overline{\varphi}(P, s) = 1/\omega \int_{F_s(P)} \varphi(Q)\,d\omega$. Le problème (6 — 8)** ainsi obtenu pour V est à deux dimensions quel que soit n. Après avoir trouvé V, on trouve

$$u(P, t) = \lim_{s\to 0} V(u], P, s, t)$$

u étant obtenu, on vérifie directement que c'est la solution cherchée.

4. On trouve la solution pour V par la méthode de Fourier — Cauchy (pour $k = -\alpha^2 < 0$). En séparant les variables $V = \left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^{-p} ST$, $p = \frac{n-2}{2}$, on obtient l'équation $T'' + \lambda^2 T = 0$ et l'équation

$$\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^2 S'' + \frac{\sin\sqrt{k}s}{\sqrt{k}}\cos\sqrt{k}s\,S' + \left[\mu^2\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right) - p^2\right]S = 0 \tag{9}$$

$$\mu^2 = \lambda^2 + kp(p+1)$$

(λ étant la constante de séparation) que nous avons étudiée dans (2) (voir la remarque au § 4). En exigeant que V soit borné pour $s=0$, on obtient la solution partielle

$$\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^{-p} J_p(\mu, s)\,[A(\mu)\cos\lambda t + B(\mu)\sin\lambda t]$$

---

* Le problème de Cauchy pour l'équation de la chaleur spécifique peut être résolu d'une manière analogue.

** Supposé que V est fini pour $s=0$ (c'est-à-dire que u est fini), le problème (6—8) a une solution unique pour $k = -\alpha^2 < 0$; en ajoutant la même condition pour $s = \pi/\sqrt{k}$, on aura sans doute une solution unique pour $k = \alpha^2 > 0$. D'ailleurs, ces conditions peuvent être affaiblies.

En appliquant le principe de superposition, nous cherchons pour V une solution satisfaisant (7) sous la forme

$$V(u], P, s, t) = \int_{\frac{a}{2}}^{\infty} \left(\frac{\operatorname{sh} \alpha s}{\alpha}\right)^{-p} J_p^-(\mu, s)\, B(\mu) \sin \sqrt{\mu^2 + \alpha^2 p(p+1)}\, t \; d\mu \qquad (10)$$

La fonction $B(\mu)$ se trouve de (8) au moyen de l'une des formules d'inversion indiquées dans ([2]). Nous obtenons finalement:

$$V = \left(\frac{\operatorname{sh} \alpha s}{\alpha}\right)^{-p} \int_{a/2}^{\infty} K_p(\mu) \frac{\sin \sqrt{\mu^2 + \alpha^2 p(p+1)}\, t}{\sqrt{\mu^2 + \alpha^2 p(p+1)}} J_p^-(\mu, z)\, \omega(P, \mu)\, d\mu \qquad (11)$$

$$\omega(P, \mu) \equiv \int_0^{\infty} \left(\frac{\operatorname{sh} \alpha z}{\alpha}\right)^{p+1} J_p^-(\mu, z)\, \bar{\varphi}(P, z)\, dz$$

A l'aide de la formule récurrente (5_1) ([2]), de la répresentation intégrale pour $J_0^-(\mu, z)$ (qui résulte de ce que (5_2) ([2]) subsiste quand $m \geqq 0$ n'est pas un entier), nécessaire quand on considère le cas $n = 2m$, et de l'intégrale connue de Sonine ([3]), en faisant le passage à la limite pour $s \to 0$ dans (11), on arrive aux solutions:

$$u = \frac{1}{2m-1!!} \int_0^t J_0(m\alpha \sqrt{t^2 - z^2}) \frac{d}{dz} L_{m-1}\left[\left(\frac{\operatorname{sh} \alpha z}{\alpha}\right)^{2m-1} \bar{\varphi}(P, z)\right] dz \qquad (12_1)$$

pour $n = 2m+1$, et

$$u = \frac{1}{m-1!! 2^{m-\frac{1}{2}}} \int_0^t \operatorname{sh} \alpha z\, R_m(z, t, \alpha)\, L_{m-1}\left[\left(\frac{\operatorname{sh} \alpha z}{\alpha}\right)^{2m-2} \bar{\varphi}(P, z)\right] dz \qquad (12_2)$$

pour $n = 2m$, où

$$R_m(z, t, \alpha) = \frac{1}{\sqrt{\operatorname{ch} \alpha t - \operatorname{ch} \alpha z}} - \int_z^t \frac{\partial J_0\left(\frac{2m-1}{2} \alpha \sqrt{t^2 - \tau^2}\right)}{\partial \tau} \frac{d\tau}{\sqrt{\operatorname{ch} \alpha \tau - \operatorname{ch} \alpha z}}$$

qui sont réunies dans (5) pour $k = -\alpha^2$.

Remarque. Notons ici que l'équation (9) se réduit à l'équation de Legendre si l'on fait la substitution $\sqrt{ks} = x$. Cette dernière équation a été l'objet d'études détaillées en XIX siècle. Ainsi d'abord elle a été considérée pour $p = 0$ et $\mu^2 = kn\,(n+1)$, où n est un entier; ensuite pour $p = 0$ et μ quelconque elle a été étudiée par Schläffli qui a introduit, ainsi les fonctions de Legendre d'indice arbitraire. Pour p entier et μ quelconque elle a été considérée par Ferrers et, enfin, pour p, μ et x quelconques par Hobson([4]) dans son grand mémoire de 1896. Le fait que la fonction de Bessel $J_p(x)$ peut être considérée comme cas limité de la fonction de Legendre-Hobson a été remarqué par maints auteurs, mais on n'en a déduit rien d'essentiel. Cependant, si l'on écrit l'équation de Legendre dans notre forme (9), avec un paramètre supplémentaire, on est conduit immédiatement à construire pour cette équation une théorie parallèle à la théorie bien développée de l'équation de Bessel à laquelle elle se réduit pour $k \to 0$. Ce point de vue est justifié, par example, par ce fait que la formule spectrale de Hankel pour les fonctions de Bessel a été donnée en 1875, et sa généralisation possible pour les fonctions de Legendre-Hobson restait inaperçue jusqu'à présent. Nous avons indiqué dans ([2]), comment on peut se placer systématiquement à ce point de vue, et dans la même Note nous avons, en particulier, donné pour la première fois les formules d'inversion (9) ([2]); remar quons, en passant, que les formules d'inversion, trouvées par W. Fock et qui font l'objet de sa Note récente ([7]), présentent un cas particulier pour $p = 0$ de nos formules (9) ([2]), puisque

$$J_0(\mu, s) = P_\nu(\cos \sqrt{k}\, s), \quad \nu = \frac{1}{2} + \sqrt{\frac{\mu^2}{k} + \frac{1}{4}}.$$

D'ailleurs, il est facile à voir que les conditions du théorème 3 de la Note ([7]) de W. Fock sont plus faibles que nos conditions, tandis que les conditions du théorème 2 de W. Fock pour l'inversion duale sont, au contraire, plus fortes que les conditions correspondantes de notre théorème.

5. Pour $k=\alpha^2$ en vertu de ce que dans ce cas $0 \leqslant s \leqslant \pi/\sqrt{k}$, en exigeant que V soit fini pour $s \to \pi/\sqrt{k}$, on obtient un spectre discret de valeurs de μ (voir § 9 dans (2)): $\mu_N = \alpha\sqrt{(p+N)(p+N+1)}$, $N=0, 1, \ldots$, $p=(n-2)/2$.

En suivant la méthode de Fourier nous obtenons finalement à l'aide du § 9 (2) la solution de V dans la forme d'une série

$$V(u], P, s, t) = \sum_{N=0}^{\infty} A_N \left(\frac{\sin \alpha s}{\alpha}\right)^{-p} J_p^*(\mu_N, s) \frac{\sin \sqrt{N(N+n-1)}\,\alpha t}{\sqrt{N(N+n-1)}\,\alpha} \qquad (11')$$

$$A_N = \frac{\int_0^{\pi/\alpha} \left(\frac{\sin \alpha z}{\alpha}\right)^{p+1} J_p^*(\mu_N, z)\, \bar{\varphi}(P, z)\, dz}{\int_0^{\pi/\alpha} \frac{\sin \alpha z}{\alpha} J_p^{*2}(\mu_N, z)\, dz}$$

En tenant compte de ce que la fonction φ est supposée dérivable un nombre suffisant de fois, il est facile à démontrer à l'aide de (5) (2) et de l'intégration par parties itérée que $A_N = o(\mu_N^{-p-1})$.

En passant à la limite par rapport à s sous le signe de somme (ce qui est légitime ici), on trouve

$$u(P, t) = \sum_{N=0}^{\infty} C_N \sin \sqrt{N(N+n-1)}\,\alpha t \qquad (12')$$

$$C_N = \frac{\mu_N^p}{2^p \Gamma(p+1)\sqrt{N(N+n-1)}\,\alpha} A_N$$

Cette solution peut être représentée dans la forme (5) si l'on prolonge $\bar{\varphi}(P, z)$ pour $z > \pi/\sqrt{k}$, comme il a été indiqué dans le théorème (§ 2).

6. Pour démontrer directement que le passage à la limite dans (11) est légitime nous sommes nécessités de faire sur la fonction φ des hypothèses plus restrictives que celles qui sont exigées par la nature même du problème. C'est pourquoi pour démontrer que (5) a lieu, il faut faire une vérification directe et à cet effet s'appuyer sur les théorèmes concernant le potentiel dans R_n qui suivent des résultats de Feller (5), et sur le lemme qui affirme que $L_m J_0\left(m\sqrt{k(z^2-t^2)}\right)$ et, respectivement, $L_{m-1}R_m(z, t, \sqrt{-k})$ vérifient l'équation (2)* pour $n=2m+1$ et, respectivement, pour $n=2m$. Ce dernier découle facilement de la relation

$$\left(\frac{\partial^2}{\partial z^2} + 2m\sqrt{k}\operatorname{ctg}\sqrt{k}z\frac{\partial}{\partial z}\right) L_m = L_m\left(\frac{\partial^2}{\partial z^2} + m^2 k\right)$$

(et d'une relation analogue pour $n=2m$) que l'on démontre par induction (ici, comme dans § 4, l'opération L est effectuée par rapport à la variable z).

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* Il est nécessaire de tenir compte de l'expression $\Delta_2 u$ en coordonnées géodésiques polaires.

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MECHANICS

PENETRATION OF A THIN PLATE INTO A VISCOUS MEDIUM

By N. A. SLEZKIN

(Communicated by L. S. Leibenson, Member of the Academy, 12.V.1944)

A viscous medium is supposed to fill up the half-space below a horizontal plane X_1OZ_1. Into this medium at an instant $t=0$ a thin plate of breadth b begins to move from above with a constant velocity V. We admit the validity for a viscous medium of the Newtonian law

$$\tau = \mu \frac{\partial v}{\partial x}$$

where μ is the coefficient of viscosity; v is the projection of the velocity on the y axis, and τ is the stress due to internal friction. What we are to determine is the resistance offered by the medium to the movement of the plate.

We shall solve this problem with the aid of a movable system of coordinates having their origin at the lower edge of the plate. The directions to the right of the plate and downward are defined to be the positive directions of the x and y axis, respectively.

Denote by v the projection of the absolute velocity of a particle of the medium on to the y axis, and by u, its projection on to the x axis. Then the projection of the acceleration of this particle on to the y axis can be represented in the form

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt}$$

In the case considered the relative velocity has its projection on to the x axis exactly equal to u, that is $\frac{dx}{dt} = u$.

Its projection on the y axis, represented as the derivative of the coordinate y, can be substituted with the difference between the absolute velocity v and the translational velocity V, that is $\frac{dy}{dt} = v - V$.

So the projection of the acceleration on the y axis assumes the form

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + (v-V)\frac{\partial v}{\partial y} \qquad (1)$$

Denoting by ρ the density of particles in the viscous medium and making use of the above expression for the force of friction, the differental equation of the absolute motion of particles with respect to movable axes can be represented as a projection on to the y axis in the form

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + (v-V)\frac{\partial v}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2} \qquad (2)$$

If now the quadratic forces of inertia and the local variation of the velocity be neglected, the differential equation of motion will take the form

$$\frac{\partial v}{\partial y} = -k\frac{\partial^2 v}{\partial x^2} \tag{3}$$

where

$$k = \frac{\mu}{\rho V} \tag{4}$$

Let the following boundary conditions be imposed upon the differential equation (3)

$$\left.\begin{array}{ll} y=0 & v=0 \\ x=0 & v=V \text{ for } y<0 \\ x=+\infty & v=0 \end{array}\right\} \tag{5}$$

These conditions mean that: 1) until the edge of the plate reaches it, the medium is at rest; 2) the particles of the medium stick to the lateral surfaces of the plate and, finally, 3) the velocity of the particles decreases away from the plate down to zero.

Obviously, the evolution of equation (3) has no sense unless for the negative values of y. Therefore put $y=-s$.

The solution of equation (3) under conditions (5) is well known. For the right quadrant it has the form

$$v = \frac{2V}{\sqrt{\pi}} \int\limits_{x/2\sqrt{ks}}^{\infty} e^{-\beta^2} d\beta = V\left(1 - \frac{2}{\pi}\int\limits_0^{\infty} e^{-a^2 s} \sin\frac{ax}{\sqrt{k}}\frac{da}{a}\right) \tag{6}$$

Differentiating the left and right member of (6) with respect to x and putting then $x=0$, we obtain for the force of friction τ at the plate the expression

$$\tau = -\frac{\mu V}{\sqrt{\pi k s}} \tag{7}$$

It follows from this formula that at the point at which the edge of the plate meets the viscous medium, the force of friction becomes infinite for $s=0$, which is quite legitimate.

Substituting in (7) the value of k from (4), we obtain the dependence of the force of friction on the velocity V and the position, which concides with the well-known relation in Prandtl's theory of the laminar boundary layer:

$$\tau = -\frac{\eta V^{3/2}}{\sqrt{\pi \nu s}} \tag{8}$$

In order to obtain the total resistance F, we multiply the two parts of this relation by $2bds$ and integrate from 0 to h, the length of the submerged portion of the plate. As a result, we shall obtain the following formula for resistance to the penetration of a thin plate into a viscous medium

$$F = \frac{4b}{\sqrt{\pi}} \sqrt{\mu\rho h}\, V^{3/2} \tag{9}$$

It will be obvious that the resistance offered by a viscous medium to a thin plate cutting into it depends not merely on the velocity of penetration V, but on the depth of penetration h also.

We may adopt the relation (9) when the plate is supposed to move into the medium at a variable speed. It will then be possible to study the penetration due to a permanent force or to an impact.

Suppose some weight to be added to the plate, which makes up together with the plate a total weight P. Then the differential equation of the

submergence of the plate takes the form

$$\frac{P}{g} V \frac{dV}{dh} = P - \frac{4b}{\sqrt{\pi}} \sqrt{\mu \rho h}\, V^{3/2} \qquad (10)$$

This equation can be solved, for instance, by the method of successive approximations. If the conditions of the experiment make it possible to neglect the force P, we obtain the following formula upon the separation of variables in (10) and integration:

$$\sqrt{V_0} - \sqrt{V} = \frac{4g}{3\sqrt{\pi}} \frac{\sqrt{\mu \rho}}{P} b h^{3/2} \qquad (11)$$

By means of this formula the viscosity coefficient of the medium μ can be determined from the initial and final velocity and from the distance h made by the plate in the medium. In this way the Charpy pendulum hammer can be adapted for measurements of the coefficient of viscosity μ, when the viscous medium is too solid to allow the use of the ordinary methods of viscosity determination in liquids.

It must also be borne in mind that the viscosity coefficient, as defined by Newton, and the impact viscosity of metals, as determined from the failure of notched samples, have nothing in common. Whereas, for instance, the dimension of the impact viscosity is the ratio of the work to the cross-section area, the dimension of the coefficient of viscosity, according to Newton, is the product of the specific work dimension by second. Moreover, the impact viscosity is a comparative characteristic of the behaviour of different metals under equal conditions of test. On the other hand, the viscosity defined according to Newton is not merely a comparative characteristic, but to a certain degree an absolute one in the sense in which the viscosity of a liquid is. And we have to take account of this characteristic expressly in such cases where for a given medium under given conditions not the deformations themselves are of importance, but rather their velocities.

Let us note further that our reasoning and computations are also valid in the case of visco-plastic media of Binghame-Iljushin, for which the law of friction assumes the form

$$\tau = k + \mu \frac{\partial v}{\partial n} \qquad (12)$$

where k is the plastic constant of the medium. Even the second boundary condition (5) holds good, with the sole difference that litteraly it will no longer express the sticking of a particle to the plate, but merely the fact that the particles adjoining the plate must at any particular instant have their velocities equal to the velocity of the plate. It will be apparent, however, that this condition does not mean that the particles having acquired a certain velocity at one instant cannot lose it at the next completely or partly. What is possible for the velocities may prove to be impossible for the displacements. The necessity of making this distinction is justified not only in the classical theory of impact, but also by the facts of mutual transformation of different energy forms.

For a visco-plastic medium the differential equation (10) assumes the form

$$\frac{P}{g} V \frac{dV}{dh} = P - kbh - \frac{4b}{\sqrt{\pi}} \sqrt{\mu \rho h}\, V^{3/2} \qquad (13)$$

This equation can be solved by various approximate methods, which will again afford the possibility of using the Charpy machine in determining the viscosity according to Newton for visco-plastic media.

The further development of the theoretical calculations and experimental studies outlined here will probably make it possible to shed additional light upon at least some of the obscure points in the domain of utilization of metals, in which the deformation velocities come more and more into prominence.

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TECHNICAL PHYSICS

BAND HEADS OF AlO SPECTRUM AS AN INNER STANDARD IN THE SPECTRUM ANALYSIS OF ALUMINIUM ALLOYS

By G. G. POLOZOVA and N. S. SVENTITSKY

(Communicated by A. N. Terenin, Member of the Academy, 11.V.1944)

The spectrum analysis of aluminium alloys is greatly handicapped by the scarcity of lines in the aluminium spectrum. In a number of cases the investigator finds himself unable to determine certain elements, because of the failure of his attempts to select comparison lines of aluminium, especially in the visible region of an arc discharge spectrum. When a visual method is employed in the analysis, we have either to give up the estimation of certain admixtures, say, copper, manganese, magnesium, or to use as the second electrode rods of iron and copper with whose lines the lines of the elements under determination can be compared.

The estimation of copper must be carried out with an iron electrode. This requires the use of a spectral devise of high dispersion, whereas the analysis of nonferrous alloys can be made with the aid of a spectroscope, provided with a medium-size prism. Considerable interest attaches to the possibility of copper estimation by means of such apparatus. It is known that in the wave-length region of 4000—5600 Å lies the molecular spectrum of AlO ($^2\Sigma - {}^2\Sigma$) with the band heads of which we can compare the closely located spectrum lines of the elements concerned. Table 1 is a scheme of band heads in the AlO spectrum. It may be inferred from it that in order to estimate copper and magnesium, we can compare copper lines λ 5105, 5153, 5218, and magnesium lines λ 5183, 5172, 5167, with the band heads (0—1) λ 5079, (1—2) λ 5102, (2—3) λ 5123,...., (7—8) λ 5205. Similarly in estimating manganese and zinc from the lines Mn λ 4754, 4823; Zn λ 4680, 4722, 4810 we can use the band heads (1 — 0) λ 4648, (2—1) λ 4672, (3—2) λ 4694,..., (9—8) λ 4801.

Table 1

Band Heads of AlO Spectrum[1]

p / n	0	1	2	3	4	5	6	7	8
0	4842	5079	5337	5615	—	—	—	—	—
1	4648	4866	5102	5358	5635	—	—	—	—
2	4470	4672	4888	5123	5376	5651	—	—	—
3	4308	4494	4694	4909	5143	5394	5666	—	—
4	4158	4330	4516	4715	—	5161	5409	5677	—
5		4181	4352	4537	4736	—	5177	5422	5687
6	—	—	4202	4373	4557	4754	—	5192	5434
7	—	—	—	4223	4394	4576	4771	—	5203
8	—	—	—	—	4243	4412	4594	4788	—
9	—	—	—	—	—	4261	4430	4610	4801
10	—	—	—	—	—	—	4279	4463	4625

This assumption has been verified by an approximate estimation of the content of copper and manganese in aluminium alloys with the aid of a styloscope. As a source of light an a. c. arc was used (2). It was located between the tested sample and the electrode made of pure aluminium(containing less than 0.05 per cent of copper and manganese). Current, 4—5 amperes. Air gap. about 2 mm. Some spectroscopical indices obtained in this way for the estimation of copper and manganese in aluminium alloys are given in Table 2.

Table 2

Spectroscopic Indices for Approximate Estimation of Copper and Manganese in Aluminium Alloys

Content of Cu, %	Intensity ratio of spectral lines	Content of Mn, %	Intensity ratio of spectral lines
0.15—0.20	Cu λ 5105 = AlO λ 5102	0.10—0.15	Mn λ 4754 ⩾ AlO λ 4694
0.3	Cu λ 5153 = AlO λ 5161	0.20	Mn λ 4754 > AlO λ 4694
2.0—2.5	Cu λ 5220 < AlO λ 5102		Mn λ 4783 ⩽ AlO λ 4788
	Cu λ 5153 > AlO λ 5143	0.3—0.4	Mn λ 4783 ⩾ AlO λ 4788
3.5—4.0	Cu λ 5220 = AlO λ 5143		Mn λ 4761 < AlO λ 4694
4.5—5.0	Cu λ 5220 ⩾ AlO λ 5102	1.0—1.20	Mn λ 4739 = AlO λ 4694
		1.50	Mn λ 4765 = AlO λ 4694

On the basis of samples of known chemical composition these indices can be established for other compositions, and the method can be extended to include the estimation of magnesium, zinc and nickel.

It seemed interesting to explore the possibility of quantitative estimations with a stylometer by comparing the spectral lines of the elements to be identified with the band heads of the AlO spectrum. In testing the fitness of AlO band heads as an inner standard for quantitative estimations, the intensity ratios of the band heads λ 5079 and λ 5102 were repeatedly measured in samples of different chemical composition.

The results of these determinations are given in Table 3. They testify to a fairly good reproducibility of the measurements of $tg^2\,\alpha$, which is the quantity characterizing the intensity ratio of the band heads (3). They also show that these intensities are independent of the composition of the sample.

Table 3

Reproducibility of Data on the Intensity Ratio of Band Heads AlO λ 5079 and AlO λ 5123

Sample	Chemical composition, %						Rotation angle of analyser (α)	$tg^2\alpha$	Probable error of a single determination of $tg^2\alpha$ in %
	Cu	Si	Fe	Mg	Mn	Zn			
2	0.06	12.16	0.32	—	—	—	50.30	1.451	0.94
7	11.6	0.59	1.03	0.16	0.49	—	50.31	1.451	1.40
8	8.54	0.11	0.28	—	—	0.53	50.25	1.444	0.94

In estimating the content of copper with a stylometer the Cu line λ 5153 was compared to the band head AlO λ 5079. A calibrated curve (see figure) was plotted on the basis of the data given in Table 4. The errors involved in the determinations of copper according to this graph are as follows: for a 5 per cent content of copper the probable error is 2.8 per cent; for a 10 per cent content, 8.5 per cent. The error increases with the content of copper, probably because the estimation of $tg^2\alpha$ is less accurate when the rotation angle of the analyser is large. The polarization photometer is known to give the least error when the rotation angle of the analyser is close to 45°.

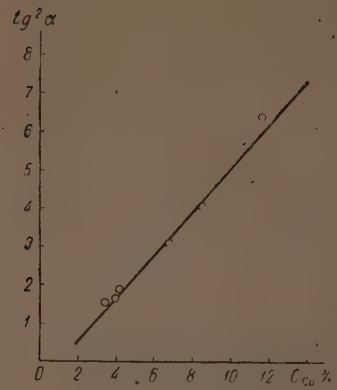

Table 4

Data Used in Plotting the Calibrated Curve for the Estimation of Copper in Aluminium Alloys

Samples	Content of copper, %	Rotation angle of analyser (α) in degrees	$tg^2\alpha$
2134	3.50	51.3	1.56
2Б	4.04	52.4	1.68
1	4.23	54.0	1.90
129	6.79	60.6	3.14
98	8.54	63.6	4.05
7	11.60	68.5	6.44

The conclusion to be drawn from the experiment is that the band heads of the AlO spectrum can be successfully used as an inner standard in the quantitative spectrum analysis of aluminium alloys for estimating the intensities of the spectral lines due to the admixture elements.

State Optical Institute.

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ELECTRICAL ENGINEERING

STABLE SPEEDS OF ROTATION OF AN ASYNCHRONOUS MACHINE WITH SINGLE-PHASE STATOR AND ROTOR

By P. A. SVIRIDENKO

(Communicated by K. I. Shenfer, Member of the Academy, 6.X.1944)

While studying the behaviour of an ordinary slip-ring asynchronous machine when one phase of its stator and rotor is broken, the author found it to possess several stable speeds of rotation. This interesting phenomenon is briefly described in the present paper.

The features distinctive of an asynchronous machine with uniaxial stator and rotor as compared to the ordinary asynchronous machine, having always more than one phase at its rotor, are these:

a) Upon uniform rotation the apparent admittance of the stator circuit varies periodically; it is a maximum when the axes of the primary and secondary windings coincide, and a minimum when they are displaced by 90 electric degrees.

b) To one period in the variation of the apparent admittance of the stator circuit (in the course of which the magnetic axes of the stator and rotor come twice

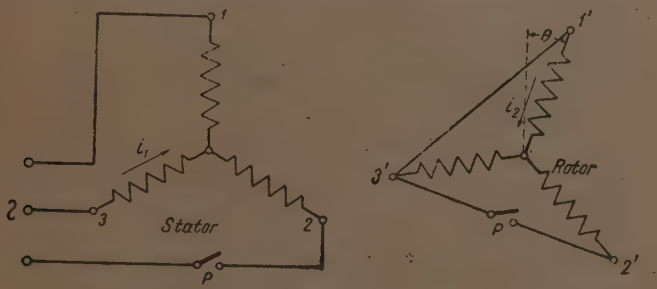

Fig. 1

into coincidence) there correspond $k = n_s/n$ periods in the voltage applied to the stator (where n_s is the synchronous, and n, the actual speed of the rotor).

c) A torque different from zero arises when the maximum (or minimum) value of the periodically varying admittance of the stator circuit is shifted with respect to the maximum (or zero) value of the variable impressed on the stator; variation in this shift should give rise to slip.

From these statements as a point of departure we set out to explain the obtained experimental results.

The experiments were run with machines of the MKA type according to the scheme represented in Fig. 1. By turning off the switch P a three-phase machine fed from a supply network at a voltage lower than nominal, was converted into

a single-phase machine with regard to both stator and rotor. The rotation speed established by this switching-off was somewhat below the asynchronous speed of the three-phase machine with the same load on the shaft. This speed proved to be only the first in a series of stable speeds. These latter were obtained by applying to the shaft of the machine a short time load of such a magnitude as to disturb the stable work at the previous speed. The load was removed (or eased) before a new stable speed of rotation was established.

In this way the following stable speeds of rotation (rev. per min.) have been obtained in a single-phase asynchronous machine working under conditions close to those of a no-load run.

Type of machine	Synchr. speed of a 3-phase machine at 50 per./sec.	Working speed	Speed of a machine with a single-phase stator and rotor						
			1	2	3	4	5	6	7
MKA-19	1000	960	941	485	315	239	191	159	133
MKA-20	1500	1470	1460	720	483	348	285	238	—

As the load at the shaft was increased, these speeds somewhat diminished*. All of this testifies to the asynchronous character of the stable speeds thus obtained. The fractional synchronous speeds corresponding to them are to the synchronous speed as

$$1,\ 1/2,\ 1/3,\ 1/4,\ 1/5,\ 1/6,\ 1/7 \qquad (1)$$

This finite series of speeds, practically limited to seven terms, can be expressed by the formula

$$n_s = 60\, f / pk_i \quad (k_i = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7) \qquad (2)$$

It has also been found by experiment that the work of a uniaxial asynchronous machine at any of these speeds is attended by electromechanical oscillations (*i. e.* by oscillation of power current, emf and rotation speed). The frequency of these oscillations turned out to be proportional to the rotor slip. They disappear when the rotation is synchronous or partly so.

Fig. 2, *a* and *b*, shows oscillograms of power and current in the stator only for two upper speeds of rotation of a uniaxial motor (the respective oscillograms of the rotor are not reproduced here).

The author has also found an analytical representation of the currents in the stator and rotor of a uniaxial motor.

In this case the differential equations have periodical coefficients. On the assumptions generally adopted in the theory of electric machines we shall get**:

$$L\frac{di_1}{dt} + Ri_1 + M\cos\theta\frac{di_2}{dt} - M\omega_p \sin\theta i_2 = \frac{U_m}{2}\cos\omega t \qquad (3)$$

$$l\frac{di_2}{dt} + ri_2 + M\cos\theta\frac{di_1}{dt} - M\omega_p \sin\theta i_1 = 0 \qquad (4)$$

where $U_m \cos\omega t$ is the voltage applied to the stator; i_1 and i_2 are currents in the stator and rotor (instantaneous values); L and l are self-induction coefficients of the stator and rotor phase; M is the coefficient of reciprocal induction between the rotor and stator; R and r are active resistances of the stator and rotor; θ is the angle between the axes of the stator and rotor windings; ω_p is the angular velocity of the rotor.

* By means of a d. c. generator installed at the shaft of the uniaxial machine the load at the shaft was brought to a value which made the work unstable. The maximum powers (torques) developed at the shaft in every zone of stable rotation proved to be rather small, as was to be expected from the design experimented upon.

** How these equations are derived, is not shown here on account of the space available.

The instantaneous torque developed by the machine (taking account of the adopted assumptions) was found to be

$$T = -2Mi_1 i_2 \sin\theta \tag{5}$$

From (3) and (4) we get the following expressions for the stationary currents of the stator and rotor:

$$i_1 = \frac{-\frac{1}{2}U_m \cos\omega t\,[l(j\omega)+r]}{M^2[(j\omega)\cos\theta - \omega_p \sin\theta]^2 - [L(j\omega)+R][l(j\omega)+r]} \tag{6}$$

$$i_2 = \frac{\frac{1}{2}U_m \cos\omega t M\,[(j\omega)\cos\theta - \omega_p \sin\theta]}{M^2[(j\omega)\cos\theta - \omega_p \sin\theta]^2 - [L(j\omega)+R][l(j\omega)+r]} \tag{7}$$

Complex quantities can be easily eliminated from (6) and (7) by ordinary transformations.

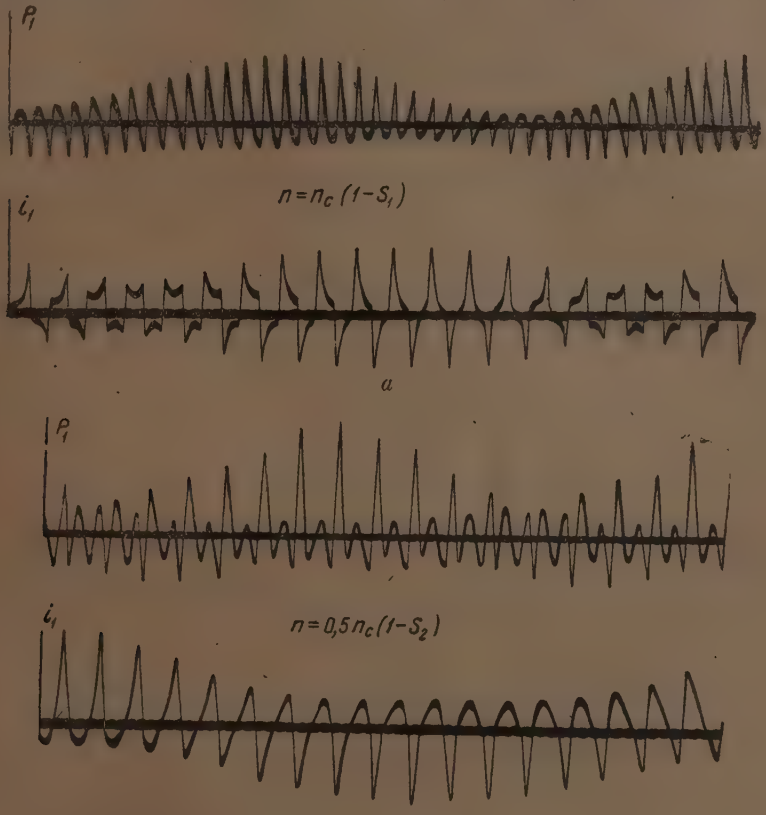

Fig. 2

Thus, for a simpler case where the active resistance of the stator and rotor is not allowed for (idealized synchronous regime) these currents are:

$$i_1 = \frac{U_m \omega l}{2M^2}\left[\frac{(\omega_p \omega \sin 2\theta)\cos\omega t + \left(\omega_p^2 \sin^2\theta - \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)\sin\omega t}{(\omega_p \sin 2\theta)^2 + \left(\omega_p^2 \sin^2\theta - \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)^2}\right] \tag{8}$$

$$i_2 = -\frac{U_m}{2M}\left[\frac{\omega_p \sin\theta\left(\omega_p^2 \sin^2\theta + \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)\cos\omega t}{(\omega_p\omega\sin 2\theta)^2 + \left(\omega_p^2\sin^2\theta - \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)^2} + \right.$$

$$\left. + \frac{\omega\cos\theta\left(-\omega_p^2\sin^2\theta - \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)\sin\omega t}{(\omega_p\omega\sin 2\theta)^2 + \left(\omega_p^2\sin^2\theta - \omega^2\cos^2\theta + \frac{Ll\omega^2}{M^2}\right)^2}\right] \qquad (9)$$

From these formulae, if the parameters are given, we can compute the rotor and stator currents as well as the torque for different speeds of the rotor in a uniaxial machine.

From the results obtained we are permitted to make the following statements.

1) The established finite series of stable asynchronous speeds of rotation, which is very nearly expressed by the ratio n_s/k_i (where $k = 1, 2, 3, 4, 5, 6, 7$), arises from the uniaxiality of the stator and rotor and is not connected with the slot harmonics of the magnetic field. These speeds are asynchronous in character.

2) The torque developed by a uniaxial induction machine in the zones of stable work is composed of a constant torque and a variable one pulsating with the slip frequency. Owing to this, the operation of the machine is accompanied by electromechanical oscillations.

The author is working further at the theory of the uniaxial induction machine, and it is his intention to consider the cases of excitation from an external supply network and from condensers.

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PHYSICAL CHEMISTRY

NUCLEAR DISTANCES AND BOND MULTIPLICITY

By B. V. NEKRASSOV

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The dependence of nuclear distances in molecules upon the multiplicity ρ of the co-valent bonds between corresponding atoms is, as is known, a qualitatively established fact. However, no one has as yet made an attempt at a uniform quantitative correlation of the relevant experimental data. At the same time with the development of the theory of resonance the problem of obtaining the effective bond multiplicity on the basis of the experimentally determined nuclear distance d becomes highly important.

This can be seen if only from the empirical Pauling curve for the particular case of resonance between the single and double bonds of carbon atoms ([1]), widely used in theoretical constructions.

If we denote the principal quantum number of an element by n and the distance between the nuclei of its atoms for a single co-valent bond ($\rho = 1$) by d_{11}, the quantitative dependence we are looking for will have the following form:

$$\rho = \left(\frac{d_{11}}{d}\right)^{(n+1)^2/2}$$

A direct check of this relation can be made on those elements for which experimentally determined nuclear distances for two or more different bond multiplicities are available. Suitable data for such a check are given in Table 1.

Table 1

Check of Equation

Element	C			P		S			Se	
n	2			3		3			4	
Substance studied	Diamond	C_2H_4	C_2H_2	P_{sol}	P_2	S_{sol}	S_2		Se_{sol}	Se_2
d_{exp} (Å)	1.542	1.328[2]	1.204[3]	2.19[2]	1.895[3]	2.10[2]	1.92±0.03[4]		2.32[2]	2.19±0.02[5]
ρ theor	1	2	3	1	3	1	2		1	2
ρ calc	—	1.96	3.04	—	3.18	—	2.05±0.26		—	2.06±0.23

As follows from Table 1, in the case of carbon, for whose initial compounds the structural parameters have been determined quite exactly (possible errors are of the order of ± 0.005 Å), the calculated values are in very good agreement with theory. The quite considerable deviation of both values of ρ for phosphorus is completely eliminated (as follows from the data for sulphur) by decreasing d_{11} by only 0.02 Å, which lies within the limits of the usual order of errors in structu-

ral determinations of a similar nature. Likewise in good agreement with theory are the calculated values for sulphur and selenium. On the whole the data in Table 1 convincingly prove the applicability of the given equation which, notwithstanding its simplicity, describes quite exactly the dependence it embraces.

Taking into consideration, at the same time, the universality of this equation, one may regard it as a reliable basis for computing co-valent atomic radii for different bond multiplicities. In an available table of such radii [2] a considerable part of the data is altogether without any basis, since it was obtained by arbitrarily decreasing the radii for a single bond (r_1) by 0.10 Å and by 0,17 Å upon passing to the double (r_2) and triple bonds (r_3), respectively. Moreover, for individual elements (O, F) the radii assumed by Pauling are in marked disagreement with the values for nuclear distances in corresponding diatomic molecules. At the same time, from the very concept of additivity of co-valent radii it is precisely such elementary molecules that should be made use of wherever possible to establish the initial values.

Table 2

Initial Data

Bond	Substance	ρ	d_{exp}	Method	r_n
C—C	Diamond	1	1.542	R[2]	$r_1=0.77$
Si—Si	Si_2H_6	1	2.32±0.03	E[3]	$r_1=1.16$
N—N	N_2	3	1.094	S[7]	$r_3=0.55$
P—P	P_2	3	1.895	S[3]	$r_3=0.95$
O—O	O_2	2	1.204	S[7]	$r_2=0.60$
S—S	S_2	2	1.92±0.03	E[4]	$r_2=0.96$
Se—Se	Se_2	2	2.19±0.02	E[5]	$r_2=1.10$
Te—Te	Te_2	2	2.59±0.02	E[5]	$r_2=1.30$
F—F	F_2	1	1.45±0.05	E[3]	$r_1=0.72$
Cl—Cl	Cl_2	1	1.983	S[7]	$r_1=0.99$
Br—Br	Br_2	1	2.28	S[7]	$r_1=1.14$
I—I	I_2	1	2.660	S[7]	$r_1=1.33$

The initial values taken as a basis in the present work are given in Table 2, in which the symbols S, E and R that have been entered opposite them indicate the methods (spectroscopic, electronographic and roentgenographic, respectively) which were employed in their determination.

The calculation of other values of r_n was made by substituting into the equation the values of ρ also indicated for the corresponding bonds. The results of the calculation are given in Table 3.

Table 3

Co-valent Atomic Radii (in Å)

ρ	C	N	O	F	Si	P	S	Cl	Se	Br	Te	I
1	0.77	0.70	0.70	0.72	1.16	1.09	1.05	0.99	1.16	1.14	1.35	1.33
2	0.66	0.60	0.60	0.62	1.06	1.00	0.96	0.91	1.10	1.08	1.30	1.28
3	0.60	0.55	0.55	0.56	1.01	0.95	0.91	0.86	1.06	1.04	1.27	1.25

Having at our disposal the data in Table 3 and basing ourselves on the concept of additivity, we may express d_{11} for a single co-valent bond between any atoms A and B by the sum of the corresponding radii: $d_{11}=r_A+r_B$. On the other hand, the exponent of the equation may be written in the form $^1/_2(n_A+1)(n_B+1)$.

Upon making these changes, the equation assumes the following convenient form for calculations:

$$\rho_{AB}=\left(\frac{r_A+r_B}{d_{AB}}\right)^{(n_A+1)(n_B+1)/2}$$

When employing the equation for calculating the effective values of p it should be borne in mind that even a small inaccuracy in the experimental value of d_{AB} (as well as in the initial radii for a single bond—r_A and r_B) affects the result quite essentially, the more so the higher the value of n for both atoms.

In conclusion it is necessary to emphasize that the proposed equation contains only rational atomic parameters. This indicates that the discovered relation has a certain inner physical meaning which has not been revealed as yet.

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PHYSICAL CHEMISTRY

ON A THEORY OF LOW-TEMPERATURE HEAT CAPACITY OF LINEAR MACROMOLECULES

By V. V. TARASSOV

(Communicated by A. N. Frumkin, Member of the Academy, 25.IV.1944)

Linear macromolecules, or main valence chains, are characterized by great bond energy and elasticity. This predetermines the high characteristic frequencies of the valence and deformation vibrations of the nuclei or groups. In hydrocarbon chains, for instance, the characteristic frequencies cannot possibly differ much from those that the C-atoms have in the diamond lattice. Owing to this, the quantum effects of the change in heat capacity and other thermodynamic functions should in the case of linear high polymers be observed at not too low temperatures.

The existing theories of heat motion in linear macromolecules ([1]) operate with the concept of equipartition of energy and thus completely neglect the low-temperature effects.

If a monatomic chain consists of N atoms linked together into a linear macromolecule (*i. e.* into a bounded unidimensional crystal), there will be $3N$ vibrational degrees of freedom in the system (corresponding to $3N$ possible types of standing, longitudinal and transverse waves).

For polyatomic chains the number of vibrational degrees of freedom will be $3nN$, where n is the number of atoms in one link. When $N \geqslant 1\,000$, one may assume that the condition

$$h(\nu'' - \nu') \ll kT \qquad (1)$$

where ν' and ν'' are the frequencies of any two neighbouring characteristic vibrations, holds with sufficient accuracy for all the characteristic frequencies.

Condition (1), as is known ([2]), gives us the right in computing the total vibrational energy and heat capacity to integrate the energies of the characteristic frequencies instead of to summate them, *i. e.* to make use of the Debye method. Replacement of the three-dimensional lattice by an elastic continuum, *i. e.* by a Debye sphere or an Ortvay ([3]) parallelepiped, is adequately realized in our case by replacing a linear molecule by an elastic rod and a two-dimensional macromolecule by an elastic membrane. The discreteness of the atomic structure is taken into account in the given cases just as was done by Debye, *i. e.* by selecting in the spectrum the elastic characteristic frequencies of the first $3N$ vibrational degrees of freedom.

At low (so-called Debye) temperatures, the individual features in the structure of the chains or two-dimensional macromolecules should even out for the same reasons as in the case of the three-dimensional theory, which for all lattices gives the same limiting law of the cubic dependence of the heat capacity upon the temperature.

Similarly, in the cases we are analysing other limiting laws are obtained: a square and a linear temperature dependence for the low-temperature change in the heat capacity of two-dimensional and linear macromolecules, respectively.

The starting point, in accordance with condition (1), is the type of the law of distribution of the characteristic vibrations with respect to frequencies. The differential of the number S in the frequency interval from ν to $\nu+d\nu$ is equal to:

a) $dS_3 = 9N\nu_{\max}^{-3}\,\nu^2 d\nu$ for a three-dimensional continuum (crystal).

Under the same assumptions (absence of dispersion) we get:

b) $dS_2 = 6N\nu_{\max}^{-2}\,\nu d\nu$ for a two-dimensional continuum (a two-dimensional macromolecule).

c) $dS_1 = 3N\nu_{\max}^{-1}\,d\nu$ for a unidimensional continuum (a linear macromolecule).

The internal energy u_m and the heat capacity c_m may, on the basis of (b) or (c), be expressed through the mean energy of one characteristic vibration

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

as follows:

$$u_m = \int_0^{\nu_{\max}} \bar{\varepsilon}\, dS_m$$

where m is 1, 2 or 3 for (a), (b) and (c), respectively.

Besides the already mentioned difference in the laws of distribution of the characteristic frequencies [(b) or (c) instead of (a)], we also take into account the macromolecular state, i. e. the limitation of the system examined (a macromolecule) by the number of degrees of freedom which, though large, is incomparably smaller than the Avogadro number ($N \geqslant 1\,000$). This is done by introducing along with the maximum (Debye) frequency $\nu_{\max}$, a minimum or «macromolecular» frequency $\nu_{\min}$, *i. e.* the «frequency of the fundamental tone». On this assumption all $3N$ vibrational degrees of freedom of a macromolecule, or one gram-atom of macromolecules, are found to lie within the frequency $\nu_{\min} - \nu_{\max}$. Accordingly the distribution laws (a), (b) and (c) should now be written in the following form:

$$dS_m = \frac{3mN}{\nu_{\max}^m - \nu_{\min}^m}\,\nu^{(m-1)}\,d\nu \qquad (2)$$

The expression for the internal energy then becomes:

$$u_m = \int_{\nu_{\min}}^{\nu_{\max}} \bar{\varepsilon}\, dS_m = \int_0^{\nu_{\max}} \bar{\varepsilon}\, dS_m - \int_0^{\nu_{\min}} \bar{\varepsilon}\, dS_m \qquad (3)$$

Upon making the usual substitutions: $h\nu/kT = x$, $h\nu_{\max}/k = \theta$ and $h\nu_{\min}/k = \theta_{\min}$ and introducing the Planck expression for $\bar{\varepsilon}$ we get the following generalized equation for the internal energy:

$$u_m = 3mR\,\frac{T^{(m+1)}}{\theta^m - \theta_{\min}^m}\left[\int_0^{\theta/T} \frac{x^m dx}{e^x - 1} - \int_0^{\theta_{\min}/T} \frac{x^m dx}{e^x - 1}\right] \qquad (3')$$

For high-molecular chains the degree of polymerization $z = \nu_{\max}/\nu_{\min} = \theta/\theta_{\min}$ is a big number. On this ground we may assume that even in the Debye temperature interval all frequencies from 0 to $\nu_{\min}$ would be excited completely. This gives us the right to expand in series the denominator in the second integral of equation (3′) and to discard everything but the first two members of the expansion. Upon doing this we get:

$$u_m = 3mR\,\frac{T^{(m+1)}}{\theta^m - \theta_{\min}^m}\int_0^{\theta/T}\frac{x^m dx}{e^x - 1} - 3RT\,\frac{\theta_{\min}^m}{\theta^m - \theta_{\min}^m} \qquad (4)$$

On the basis of equation (4) the generalized expression for the heat capacity assumes the form:

$$c_m = 3m(m+1)R\,\frac{T^m}{\theta^m - \theta_{\min}^m}\int_0^{\theta/T}\frac{x^m dx}{e^x - 1} -$$

$$-3mR\,\frac{\theta^m}{\theta^m - \theta_{\min}^m}\left(\frac{\theta}{T}\right)\frac{1}{e^{\theta/T} - 1} - 3R\,\frac{\theta_{\min}^m}{\theta^m - \theta_{\min}^m} \qquad (5)$$

In the particular case of a three-dimensional lattice, *i. e.* for $m = 3$, and a very large number of degrees of freedom, the quantity $\theta_{\min}$ in the difference is small in comparison with θ and may be discarded, and equation (5) reduces to the usual Debye equation for heat capacity:

$$c_3 = 36R\left(\frac{T}{\theta}\right)^3\int_0^{\theta/T}\frac{x^3 dx}{e^x - 1} - 9R\left(\frac{\theta}{T}\right)\frac{1}{e^{\theta/T} - 1}$$

The heat capacity of chains with a very large number of degrees of freedom will, on the basis of equation (5), be given by the formula:

$$c_1 = 6R\left(\frac{\theta}{T}\right)\int_0^{\theta/T}\frac{x\,dx}{e^x - 1} - 3\left(\frac{\theta}{T}\right)\frac{1}{e^{\theta/T} - 1} \qquad (6)$$

We computed the function (6) with the aid of a very simple auxiliary method based on employing the table of Einstein functions for the heat capacity. The data obtained are given in Table 1.

Table 1

Function c_1 (catena) and Function c_3 (Debye)

θ/T	T/θ	c_1	c_3	θ/T	T/θ	c_1	c_3	θ/T	T/θ	c_1	c_3
0.0	∞	5.955	5.955	1.6	0.625	5.351	5.26	7	0.143	2.747	1.137
0.2	5.00	5.937	5.94	1.8	0.556	5.458	5.09	8	0.125	2.424	0.823
0.4	2.50	5.915	5.91	2.0	0.500	5.359	4.918	9	0.111	2.171	0.604
0.6	1.67	5.889	5.85	2.5	0.400	5.194	4.45	10	0.100	1.949	0.452
0.8	1.25	5.847	5.77	3	0.333	4.812	3.948	12	0.083	1.630	0.267
1.0	1.00	5.792	5.670	4	0.250	4.173	2.996	14	0.071	1.357	0.169
1.2	0.833	5.726	5.55	5	0.200	3.618	2.197	16	0.063	1.226	0.113
1.4	0.714	5.649	5.41	6	0.167	3.136	1.582	20	0.050	0.978	0.058

The curve for c_1 lies above that for c_3 and, as distinct from the latter, has no point of inflection, but at low temperatures approaches a straight line as a limit (see figure)*.

Analytically, the transition to low temperatures may be effected in all cases by making the upper limit of the integral in equation (5) infinite. Upon so doing this integral may be represented by the sum of a series:

$$D_m(\infty) = \int_0^{\infty}\frac{x^m dx}{e^x - 1} = \sum_{k=1}^{k=\infty}\int_0^{\infty} x^m e^{-kx} dx = m!\sum_{k=1}^{k=\infty}\frac{1}{k^{m+1}} \qquad (7)$$

The integrals we are interested in give for $m = 1$, 2 and 3:

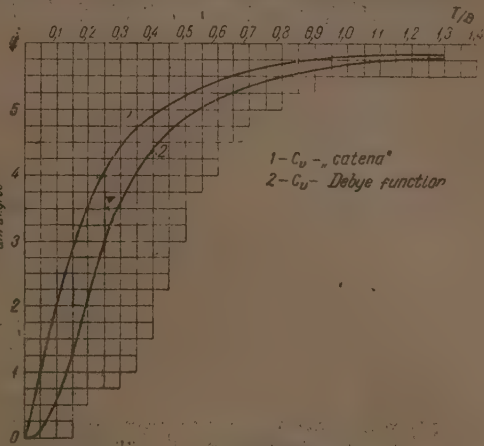
$$D_1(\infty) = \pi^2/6 \cong 1.645;\quad D_2(\infty) \cong 2.404;\quad D_3(\infty) = \pi^4/15 \cong 6.494 \qquad (8)$$

* In the figure the function c_1 stands for c_t—catena (the Latin for «chain»), and c_3 for c_v—Debye.

On the basis of equation (5) and the value of $D_1(\infty)$ from (8) we may write the expression for the limiting law of the heat capacity of high-molecular chains. Disregarding θ_{min} in comparison with θ and taking into account that $\theta/\theta_{min} = z$, *i. e.* the degree of polymerization, we may write this expression in the following form:

$$c_1 = \pi^2 R \left(\frac{T}{\theta}\right) - \frac{3R}{z} \text{ (per gram-atom)} \qquad (9)$$

Equation (9) shows that the limiting law for c_1 is given by a straight line with a negative intercept $A = 3R/z$, which we term the «macromolecular factor». The macromolecular factor is a unique thermodynamic criterion of the macromolecular weight, its magnitude being inversely proportional to z.



It should be emphasized that the above extrapolation method of obtaining A or z (as a negative intercept) does not contain any confirmation of how the curve actually approaches the absolute zero. The point is that the present theory, just as the usual theory of Debye, holds for low but not super-low temperatures. The reasons for this, inherent in condition (1), have already been examined by Planck [2].

At super-low temperatures, when condition (1) is not fulfilled, we have no right, as has already been pointed out, to replace the sums by integrals, and hence we cannot draw any conclusions from the Debye theory.

This, in conjunction with what we have said at the beginning of this paper, sets the limits of application of the limiting laws of the Debye type for both high and low temperatures.

Table 2

Type of continuum	u_m	c_m
Unidimensional	$u_1 = \frac{\pi^2}{2} \frac{R}{\theta} T^2 \simeq 4.935 \frac{R}{\theta} T^2$	$c_1 = \pi^2 R \frac{T}{\theta} \simeq 9.87 R \frac{T}{\theta}$
Two-dimensional	$u_2 = 14.42 \frac{R}{\theta^2} T^3$	$c_2 = 43.27 R \left(\frac{T}{\theta}\right)^2$
Three-dimensional	$u_3 = \frac{8}{5} \pi^4 \frac{R}{\theta^3} T^4 = 58.45 \frac{R}{\theta^3} T^4$	$c_3 = \frac{12}{5} \pi^4 R \left(\frac{T}{\theta}\right)^3 = 233.8 R \left(\frac{T}{\theta}\right)^3$

In Table 2 are given formulae for the limiting laws for the internal energy and heat capacity obtained from equations (4), (5) and (8) for the case of a large number of degrees of freedom.

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GEOLOGY

ON METAMORPHOSED MANGANESE DEPOSITS*

By A. G. BETEKHTIN

(Communicated by W. A. Obruchev, Member of the Academy, 19.IV.1944)

1. One of the authorities on ore deposits K. I. Bogdanovich (1) was the first to separate out with the utmost distinction within the class of metamorphic deposits the subclass of metamorphosed deposits as distinguished from the metamorphic ones in the strict sense. And yet he did not deny that «to explain the origin of metamorphosed deposits was one of the most difficult problems in our science».

Nearly a quarter of a century has elapsed since the time when these lines were written, and we witness an approach to the solution of this most important problem.

In 1936, 1941 and 1943 appeared papers of A. N. Zavaritzky (2, 4) dealing with this problem. In studying the copper-pyrite deposits of the Urals he showed the strong influence exerted by regional metamorphism not only upon the rocks enclosing the ore bodies, but also on the shape of the deposits and their composition. Contrary to all previous conceptions, these conclusions necessitate a new way of organizing the search for these deposits in the Urals.

Nearly at the same time with the work of A. N. Zavaritzky the present author in 1936, and particularly during the National war, carried out a study of the phenomena of metamorphism in manganese deposits occurring among Palaeozoic strata, metamorphosed to a various degree, in the Urals and Central Kazakhstan.

2. Nearly all of the deposits of Central Kazakhstan are represented by typical bedded bodies of manganese and iron ores. From the general geological setting they do not essentially differ from the non-metamorphosed sedimentary deposits of Tertiary age: 1) all of them are confined to bedded, folded formations composed of shallow-water siliceous chemical sediments, chiefly gaizes and, to a lesser extent. *Radiolaria* jaspers of Lower Carboniferous age; 2) in the ore-bearing strata the same facial alternations of oxide and carbonate ores are observed.

As distinguished from non-metamorphosed sedimentary deposits, their ores are composed of anhydrous crystalline oxides of manganese: braunite and hausmannite (and in iron ores, hematite and magnetite). In a number of deposits among the ore-bearing horizons and the enclosing rocks the attention is attracted by transverse veinlets, in their external appearance quite undistinguishable from typical hydrothermal structures. However, a careful study of the conditions of their occurrence reveals a highly characteristic peculiar feature: the chemical and mineralogical composition of these veinlets is intimately related to the composition of the enclosing rocks or ores. Thus, *e. g.*, in dense siliceous rocks, quartz veinlets are developed; in grey limestone bands, white veinlets of calcite; in highly ferruginous jaspers, quartz veinlets with lamellar crystals of hematite; in layers of braunite and hematite ores, veinlets of the same minerals, *etc.* It is highly chara-

* A paper read before the Division of Geological and Geographical Sciences of the Academy of Sciences of the USSR at a meeting held on the 60th anniversary of A. N. Zavaritzky, Member of the Academy.

cteristic too that in these veinlets no such chemical elements and minerals have been established that would indicate their genetic relation to the hydrothermal activity of acid magma intrusions (gold, silver, copper, lead, zinc, antimony, *etc.*). Neither has it been possible to establish their spatial association with any intrusive masses of igneous rocks.

3. All these features have much in common with veins of the Alpine type, formed in the process of regional metamorphism of rocks without any connexion with magmatic processes whatever. These veins are known to be characterized by the fact that the walls of the hollow fractures, originating in the process of rock deformation, are covered with druses of crystals of those minerals which are formed in the course of metamorphism in the enclosing rock itself.

As distinguished from veins of the Alpine «cleft» minerals, our veinlets are, as a rule, completely filled up with mineral matter. In addition, a wedging of these veinlets into rocks of an alien composition is frequently observed. The latest in time of segregation are barite and calcite, occasionally as independent veinlets. Rare sulphides of the elements contained in the rocks themselves occur not infrequently, too. Finally, metasomatosis phenomena are of no rare occurrence.

4. In the rearrangement of the mineral matter in the process of metamorphism, the chief part is played by H_2O , CO_2 and other mineralizers contained in the surrounding rocks or partly supplied from outside. The degree of filling of the fractures with mineral matter depends on the number of agents taking part in the mineralization.

If it be considered that the sedimentary rocks and the ores of iron and manganese up to the moment of metamorphism were represented in their bulk by hydroxides (opal, limonite, psilomelane, *etc.*), it will be clear that in the case of regional metamorphism in the process of formation of anhydrous oxides a huge amount of water should be liberated. Under conditions of elevated temperature and pressure this liberated water should acquire the properties of typical magmatic hydrothermae, in particular, a higher capacity to dissolve, transport and deposit mineral matter in voids. It is not surprising therefore that closed up fractures, originating under the action of stress, would become completely filled up with this substance. The more readily soluble compounds, being deposited on the latest stages of the process, would, naturally, be concentrated within a certain distance from the original source.

5. It goes without saying that the most favourable conditions for a circulation of metamorphic waters exist along the fault disturbances and crushing zones. There is, therefore, nothing surprising in the fact that along such zones, among siliceous rocks, outcrops of rather thick veins of «barren» quartz, *i. e.* quartz containing no ore, and of other minerals, containing neither gold, nor valuable metals, are not infrequently observed. It has not been possible spatially to associate such veins with intrusions of acid igneous rocks.

If such a fault passes through a bedded deposit of sedimentary rocks, in the process of metamorphism a partial redeposition of the ores themselves may also take place. Evidently, in external appearance these redeposited ores will differ little from typical ores of hydrothermal origin. There is every reason to expect such an instance at the Djezdy manganese deposit (Central Kazakhstan) prospected at present.

6. According to the redox conditions, which in turn depend on the depth at which metamorphism takes place, we observe different mineral associations in metamorphosed ores, the behaviour of manganese being highly characteristic. At shallow depths, with a sufficient supply of oxygen in the process of metamorphism, abundant veinlets of evidently crystalline pyrolusite are extensively developed, *i. e.* compounds of Mn^{+4} (Akkerman deposit in the South Urals). At greater depths the ores are converted to braunite-hausmannite masses (Central Kazakhstan). Finally, under reducing conditions of a very strong metamorphism at relatively great depths we observe silicates of Mn^{+2}—rhodonite, bustamite, spessartine, *etc.* (deposits in Devonian jaspers in the region of the town of Magnitogorsk) (6).

7. In comparing the hydrothermal veinlets formed during regional metamorphism, with hydrothermal magmatic bodies resembling them, the following conclusions will be reached.

a) Veinlets in metamorphosed rocks and ores formed by deriving matter from the surrounding masses are characterized by a «sterile» mineralogical composition. Veinlets of a hydrothermal magmatic origin, as a rule, do not show such a relationship, and the regions of their development are «contaminated» with heavy metals carried in from magmatic foci (such are, *e. g.*, the regions of gold-bearing placers, the gold of which is derived from numerous disseminated small veinlets of quartz).

b) Redeposited vein masses of manganese ores in zones of fault disturbances, in addition to the sterility of composition, are characterized by a striking persistence of the type of chemical compounds, and a uniformity of the structural and textural peculiarities, while for the vein and metasomatic deposits of manganese ores, genetically related to magmatic hydrothermae, we often observe a variety not only of structures, but also of the composition of the ores. Thus, for instance, a typical hydrothermal deposit of manganese—the Sapalskoe (in the North Urals)—aside from containing gold, silver, lead (galena), copper (chalcopyrite), pyrite, is also characterized by the fact that manganese enters into the composition of the minerals now in the bivalent (rhodochrosite, manganocalcite), and now in the trivalent form (braunite, hausmannite) and, finally, as tetravalent (pyrolusite, psilomelane). In the ores of metamorphosed deposits no such a diversity of the composition of manganese minerals is observed.

One cannot deny a possible splitting from typical hydrothermal ore bodies, uniform in composition and even free from non-ferrous and rare metals. In such cases, however, we should also have those mineral bodies from which such a splitting might have occurred.

c) All these questions are not only of scientific, but also of great practical importance. The trend and success of search work depends on a correct solution of the question of genesis. If the deposits of crystalline manganese ores are really genetically related to magmatic activity, the search for them should be carried on in the regions of exposures of igneous mother rocks; and if they are the result of regional metamorphism, they should be sought for in manganiferous formations, their stratigraphy and palaeogeography being considered.

8. In conclusion it should be noted that such phenomena as described above undoubtedly occur in all the areas of regionally metamorphosed rocks. Among the probable examples, mention may be made of the magnetite veins and vein breccias of the northern part of Krivoy Rog (Ternovsky mines), the metamorphic phenomena in the auriferous conglomerates of Witwatersrand, the so-called second phase of hypogene mineralization in the hollow fissures of the Djezkazgan cupriferous sandstones, *etc.*

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MINERALOGY

VARIATION IN PARAMETERS OF MONTMORILLONITE AS AFFECTED BY PETROLEUM

By I. D. SEDLETZKY and S. M. YUSSUPOVA

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1. Numerous investigations of the mineralogical composition of fine fractions of different clayey deposits, carried out in Uzbekistan during these four years (by the junior author), have shown that in those cases where the colloid-disperse minerals were represented by a mineral of the montmorillonite group, the values of basis interference ($d_{(001)}$) were different and varied from 10.36 to 28 Å. High values of $d_{(001)}$ were always observable in X-ray photographs of those montmorillonite clays which were found in petroliferous areas. This induced the authors to undertake special investigations in order to check up the possible dependence of high values of $d_{(001)}$ on the influence of petroleum products.

2. As the objects of study, clays occurring in the immediate proximity of the petroleum regions of Central Asia were used. One set of specimens was taken directly from oil-wells. These clays were subjected to the influence of oil, of its non-volatile components. Another set of specimens consisted of clays taken in the same petroliferous regions but beyond the boundaries of oil-wells and from horizons lying stratigraphically much higher than the former. These clays were subjected to the effect of the volatile components of oil. The third set of clay specimens was taken from unquestionably non-petroliferous regions. The effect of oil and of its components on these specimens was quite excluded.

Specimens of clays from oil regions were taken from the following deposits:

1) Changyr-Tash, 2 specimens. Specimen No. 1 soaked with oil, and specimen No. 2 taken from the gas-bearing formation.

2) Shor-Su, 2 specimens. Specimen No. 1 soaked with oil, and No. 2 taken from the gas-bearing formation.

3) Ryshtan, 1 specimen. Taken from the gas-bearing formation.

4) Tash-Ravat, 1 specimen. Taken from the gas-bearing formation.

From non-petroliferous regions, specimens were taken from the following deposits:

1) Oglandy (Djebel), 1 specimen.

2) Suluykta, 1 specimen.

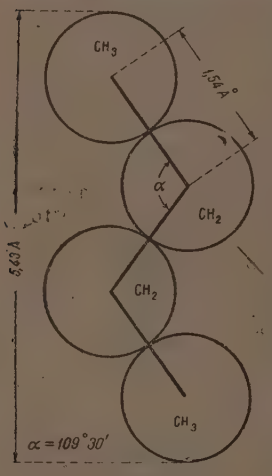
3. The first run of experiments consisted of investigations of specimens in their natural state.

The second run consisted of analyses of same specimens, but saturated with oil. The saturation was carried out by two methods: the first was to immerse the clay specimens directly into the oil and to keep them in the oil for 15 days. The second method was to saturate the clay with volatile petroleum products, to which end the specimens were placed in an desiccator over the oil and kept there at a tempera-

ture of 40—50° C until a constant weight was established. The specimen was ground in an agate mortar and subjected to X-ray analysis by the Debye-Sherrer method. Fe Kα radiation was obtained at 35 kV and 15 mA. The camera had a diameter of 114.74 mm. Exposure, 9 to 14 hours.

4. The results of the first run of experiments were as follows. The X-ray characteristics show the presence of a complex of minerals in all the clays. But at this point we are interested only in that in all the specimens minerals of the montmorillonite group are present. This follows in the first place from the basis interferences which for the minerals of this group have the value > 10 A *.

The highest values of basis interferences have been found in the Changyr-Tash specimen No. 1, for which $d_{(hkl)} = 20.24$ Å, and in specimen No. 1 from Shor-Su, with $d_{(hkl)} = 20$ Å. The No. 1 specimen from Changyr-Tash was taken from the core and had consequently been subject to the action of oil. The No. 1 specimen from Shor-Su was taken from Naphthahaus and hence was also subject to the action of oil. It is only by the adsorption of some fractions of petroleum hydrocarbons that such an extension of the crystalline lattice of montmorillonite along the c axis may be explained**.



Butane

Specimens No. 2 from Changyr-Tash, No. 2 from Shor-Su and from Ryshtan have practically the same $d_{(001)}$, 14—15 Å. The drop in the value of $d_{(001)}$ for the montmorillonite mineral of these specimens is associated with a less intensive action of oil components. All of them, although found in oil-bearing regions, are in no direct contact with oil: they lie stratigraphically much higher. Only the volatile petroleum fractions penetrate here, which, being adsorbed, cause the extension of the packets of the montmorillonite lattice along the c axis by a smaller value. These smaller values are closely related to the shorter chains of the hydrocarbons. It is known that the most readily volatile petroleum fractions, the saturated hydrocarbons, have up to 5—7 carbon atoms, up to pentane and heptane. From crystal-chemical calculations it follows that the heptane chain has a length of 9.82 Å (see figure). Upon the adsorption of the hydrocarbons by montmorillonite, $d_{(001)}$ is equal to 14—20 Å. The No. 1 Shor-Su and No. 1 Changyr-Tash specimens were in direct contact with oil, adsorbed the heavier hydrocarbon fractions and had $d_{(hkl)} = 20$ Å, and those specimens which were subjected to the effect of volatile oil components had a smaller $d_{(hkl)}$ equal to 14—15 Å.

What are exactly the hydrocarbons then adsorbed, is not certain. As oil contains different hydrocarbons, with different chain lengths reaching 70 Å, it may be assumed that many of them may be adsorbed. However, as the packets of the lattice are extended only to 20.00 Å, it may be assumed that in the case of interaction of montmorillonite and oil, some definite fractions are selectively adsorbed. Assuming the relation between $d_{(001)}$ of the lattice and the length of the chain of the hydrocarbon, it is possible to find the length of the chain of the adsorbed hydrocarbons. Taking as the initial value of the packet of montmorillonite $d = 10$ Å (for a dry specimen with the closest spacing of the packets), and deducing the value from the specimen saturated with oil, we obtain a value of 10 Å, which determines the length of the hydrocarbon chain. By means of crystal-chemical calculations this value is shown to correspond to saturated hydrocarbons—heptane (19.88 Å) and octane (11.36 Å).

* A high value of $d_{(001)}$ is possessed also by the nontronite group of minerals; however, the phenomena of extension of the lattice along the c axis for this group have not as yet been sufficiently investigated.

** The lattice of montmorillonite is capable to extend along the c axis when adsorbing water or other substances. Detailed explanation will be done later.

X-ray photographs of the remaining three clays from Tash-Ravat, Oglandy and Sulyukta give for montmorillonite $d = 10.21—10.50$ Å. All the three clays are found in non-petroliferous regions.

5. The experiments of the second run consisted in saturating two clays containing montmorillonite and taken from regions known to be non-petroliferous: the Oglandy and Sulyukta clays.

In the first run these clays were saturated with water, by placing them in an desiccator. The specimens were kept over water surface and could be saturated with water vapour only. The increase in weight amounted to 20 per cent.

The second run consisted in experiments involving the saturation of the specimens of the same clays with volatile oil fractions. The specimens were placed in a desiccator with oil and held over oil.

The third run consisted in experiments on the direct saturation with oil of clay specimens from Oglandy and Sulyukta. The specimens were kept in oil during a period of two to three weeks.

The fourth run was carried out on specimens saturated with oil (by the method described for the experiments of the third run) with a subsequent removal of the excess of oil by washing the specimens with ether.

When saturated with water vapour the clay from Oglandy increased $d_{(hkl)}$ from 10.21 to 12.30 Å. In those from Sulyukta $d_{(001)}$ increased from 10.00 to 14.37 A.

In the same clays, when saturated with volatile oil components, $d_{(001)}$ increased to 15.00 and 17 Å, respectively.

A direct saturation of these clays with oil resulted in a still greater increase of $d_{(001)}$: for the former, to 27.7 Å, and for the latter, to 27.52 Å.

In order to make sure that the new values of $d_{(hkl)}$ were not due to the oil itself, its excess was washed off with ether. The washed specimens showed on X-ray photographs $d_{(hkl)} = 20$ and 26 Å, respectively. Hence the high values of $d_{(hkl)}$ in the case of saturation of clays with oil are due to those components which have been absorbed by montmorillonite. The considerably higher values of $d_{(hkl)}$ of montmorillonite, in the case of a direct saturating of clay with oil ($d_{(hkl)} = 27.70$ Å and $d_{(hkl)} = 27.52$Å) as compared to the case of saturating with volatile components ($d_{(hkl)} = 15$ Å and $d_{(hkl)} = 17$ Å) point to different components which are adsorbed in these cases.

6. In so far as the volatile components of oil represent saturated hydrocarbons with a shorter chain than the non-volatile oil components, when adsorbing these two portions of oil with different chain lengths the montmorillonite lattice is extended to a different degree. The mechanism of this phenomenon will be discussed later. Let us but note here that the results of the experiments of the second run are in full agreement with and satisfactorily account for the results of the first run.

The different values of $d_{(hkl)}$ for the montmorillonite of clays taken from oil-bearing areas are directly dependent on the nature of the fraction of oil with which the montmorillonite is saturated. This clays which occur nearer the oil horizon adsorb the less readily volatile hydrocarbons and yield higher values of $d_{(001)}$ for montmorillonite than the clays lying stratigraphically much higher above the level of the oil. The latter clays are saturated with more volatile hydrocarbons with a shorter chain and are the cause of the lower values of $d_{(hkl)}$ for montmorillonite.

7. The mechanism of absorption of oil components by montmorillonite is as follows. Montmorillonite is known to have a crystalline lattice capable to extend in one dimension, namely along the c axis. This capacity is due to the specific structure of its lattice. Its framework is made up of packets (O—Si—O(OH)—Al (Mg,Ca)—O(OH)—Si—O) which are solidly built up of silico-oxygen and alumino-oxygen tetrahedra. Inside the packet the main valencies are operative, while between them act feeble van-der-Waals forces. Moreover, the two surfaces of the packets carry negative electric charges, repulsing the packets from one another. Owing to the existence of repulsive forces between the packets and of very feeble attractive forces water, ions and other substances can easily penetrate into the

interpacket space. This makes the packets drive apart. If more than 30 per cent water is adsorbed, the packets drive apart to a value corresponding to a $d_{(001)}$ up to 22 Å; in the case of adsorption of proteins, up to 50 Å(1).

If the adsorbed substances are removed, the packets are reversibly compressed. When the packets are driven apart, $d_{(001)}$ increases, since there exists a reverse relationship between $d_{(hkl)}$ and the Bragg angle $\sin\vartheta = n\lambda/2d$. The hydrocarbons adsorbed by montmorillonite also penetrate into the interpacket space. As a result, the packets are driven apart and increase.

What is the arrangement of adsorbed hydrocarbons between the packets? As the two internal surfaces of packets carry similar and negative charges, the arrangement of the carbon chain will be perpendicular to the plane of the packet, adjoining with their end groups $CH_3—(CH_2)_x—CH_3$, terminating with hydrogen (and hence having a positive charge), the negative charge of the surface of the packet. Hence it is clear that the longer the chain of the adsorbed hydrocarbons, the greater is the distance to which the packets are set apart, and the higher the values of $d_{(001)}$. The length of the hydrocarbon chain has been calculated by the crystal-chemical method, taking into consideration the X-ray investigation of lauric acid $CH_3—(CH_2)_{10}COOH$ by Meyer and Brill (2) who proved the zigzag shape of the chain. The angle between the two homologue groups is 109° 30′.

Moreover, the views of Pauli (3) were also taken into consideration, concerning the distribution in the zigzag molecule of electric charges which tend to attain a non-polar state.

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HISTOLOGY

ULTRAMICROSCOPIC STRUCTURE OF SPERMATOZOON

By E. KIRPICHNIKOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 4.II.1944)

The present work was made in order to study the structure of flagella of spermatozoa in polarized light by the method of a slit diaphragm. This method has been suggested by Siedentopf and developed by Szegvari (1); it has never been applied to the study of spermatozoa. The use of a polarizing microscope makes it possible to examine the finest structure of the cell. Owing to it, we are able to judge whether we have to deal with a crystalline lattice or with a gel. The particles of the latter, though isotropic, behave like anisotropic bodies because of their arrangement and shape and also because they lie less than a wave-length apart, which makes them emit light under crossed Nickol prisms.

It is not for the first time that the luminosity of flagella of spermatozoa has been studied with the aid of a polarizing microscope. Thus, Valentin (2) and Engelmann (3) who investigated the spermatozoon as a whole found it to possess double refraction. Somewhat later Mackinnon and Vles (4) declared the results obtained by them to be doubtful, pointing out the necessity to discriminate between the phenomenon of double refraction and that of depolarization. Within recent years Schmidt (5) has been studying double refraction on various tissues. On the basis of studies by Wiener (6) and Ambronn (7), Schmidt differentiated the luminescence observed into two separate phenomena: that of refraction due to shape (Formdoppelbrechung) which is the result of a peculiar shape and orientation of the mycellae, and that of double refraction proper (Eigendoppelbrechung) which always characterizes a crystalline lattice. Among the tissues analysed by Schmidt were also spermatozoa of rat, ox, salamander. The tail filament of the spermatozoa of rat and ox have been found to show typical shape-refraction, while those of the salamander display double refraction proper.

The present author studied in polarized light flagella of spermatozoa in seventeen species, *viz.*: *Paludina vivipara*, the oligopyrene and eupyrene varieties, *Planorbis planorbis* and *P. corneus*, *Limnea stagnalis* and *L. auricularia*, *Lumbricus rubelus*, gambusia, axolotl, two species of frog (*Rana temporaria* and *R. ridibunda*), *Agama* sp., *Ophiosaurus apus*, mouse, rat, cat, guinea-pig and rabbit.

Only two of them have been investigated earlier, *viz.* the frog (Valentin, 1861, and Engelmann, 1875) and the rat (Schmidt, 1937).

Thus, animals belonging to the remotest taxonomic groups were investigated, both vertebrates and invertebrates.

All the spermatozoa investigated by the author belonged to the flagellate type; the oligopyrene forms of *Paludina vivipara* were examined with the special aim to analyse the axial filament and the protoplasm separately, for which purpose they are especially convenient, having a flagellum with a clear-cut axial filament and a broad layer of protoplasm. The number of individuals examined varied as to the species from 5 (*Agama* sp., *Ophiosaurus apus*) to 40 (*Paludina vivipara*). The

flagella were examined in polarized light, under a Reichert microscope, with object-lenses of 3, 7 and an immersion system (90).

Whether the double refraction was a negative or a positive one, was determined with the aid of a gypsum plate Rot. 1. As a source of light, use was made of an Reichert arp lamp.

In order to determine the type of double refraction we selected, following the method applied by Vles, Pattry, Schmidt and others, such impregnating liquids in which the flagella ceased to be visible. The smears were dried and the spermatozoa immersed into liquids with a known refraction coefficient. The following series of liquids was applied: water, coefficient of refraction 1.33; ethyl alcohol, 1.36; glycerin, 1.47; castor oil, 1.49; ceder oil, 1.51; oleum caryophyllorum, 1.53; Canada balsam, 1.54; monobromnaphthalene, 1.66.

By the method of slit diaphragm I investigated spermatozoon flagella in nine species: *Paludina vivipara*, *Planorbis planorbis*, *Limnea stagnalis*, axolotl, *Rana temporaria*, *R. ridibunda*, cat, rat and mouse.

For lack of a ready apparatus, use was made of a dark-field condenser to which a slit diaphragm was adjusted; by rotating the latter it was possible at will to change the azimuth of illumination.

The observations gave the following results:

1. The flagella of spermatozoa of *Paludina vivipara*, *Planorbis* (2 species), *L. stagnalis* and *L. auricularia*, *Lumbricus rubelus*, gambusia, axolotl, *R. temporaria* and *R. ridibunda*, *Agama* sp., *Ophiosaurus apus*, mouse, rat, cat, guinea-pig, and rabbit when placed under crossed Nickol prisms emit a faint luminescence, *i. e.* are found to possess a slight double refraction. The degree of luminescence is about the same in all the species of spermatozoa investigated.

In gambusia it proved impossible to detect luminescence in individual spermatozoa, and entire groups of them only were found to emit a kind of faint gleam. Besides, because of the minuteness of the object it was difficult to determine whether it was emitted by the heads of the spermatozoa, or by their flagella.

In axolotl and mouse the cervical part of the spermatozoon displays a brighter luminescence than the rest of the tail filament. In axolotl spermatozoon there is an undulatory membrane along the flagellum, which remains absolutely inobservable in polarized light.

2. The flagella of the spermatozoa examined display a positive double refraction.

3. When immersed into liquids whose refraction coefficient is equal to that of the flagellum, the latter ceases to emit luminescence in polarized light. This should be taken to mean that the double refraction of the spermatozoon flagella belongs to the type of Formdoppelbrechung. The case described by Schmidt for the salamander may possibly be accounted for by an insufficiently accurate selection of the impregnating liquid, *i. e.* by a mistake in the determination of the refraction coefficient. Schmidt reports that immersion into Canada balsam resulted in no total extinction of luminescence between two crossed Nickol prisms, whereas the flagella ceased to be visible in transmitted light. According to my obsrervations, the flagella of the spermatozoa, when imbedded in Canada balsam, remained visible in transmitted light, though very faintly, and did not disappear altogether until oleum caryophyllorum was used. In the latter the luminescence of flagella ceased to be visible under crossed Nickol prisms, too.

4. The refraction coefficients of flagella in all the spermatozoa examined seem to be very close to one another, approaching that of oleum caryophyllorum, *i. e.* 1.53.

5. A detailed analysis of the double refraction of the flagella in oligopyrene types of spermatozoa of *Paludina vivipara* showed the thick layer of protoplasm in them to emit no luminescence under crossed Nickol prisms; it is the axial filament only that emits light.

6. Judging by the nature of double refraction (Formdoppelbrechung) it can be concluded that the flagella must have a clearly pronounced orientation of the mycellae, and that this orientation holds for the fibrillary structures of the flagellum only, whereas the plasm displays no signs of it.

7. The flagella of the spermatozoa investigated by the method of slit diaphragm were found to be extinguished fully as soon as the diaphragm was turned so as to make the rays fall along the length of the spermatozoon. On the contrary, they remained fully illuminated when the rays were arranged perpendicular to the length of the spermatozoon.

Following Szegvari's reasoning, I suppose that the mycellae of a spermatozoon flagellum must be arranged in the shape of a chain; they are strictly oriented, so that when light falls upon the cross-section of a mycella which is less than a micron, there can be no illumination in a dark field. If light falls upon a mycella in its longitudinal direction, the flagella look brightly illuminated.

8. If we compare our observations with data available in literature, it seems reasonable to assume the definite orientation of the mycellae to be a sign of the high specialization of the spermatozoon. This specialization must have proceeded in all animal species in a much similar way, owing to the similarity of the conditions under which the spermatozoon developed and went on functioning.

9. The data obtained demonstrate that the contractile elements of the sperm differ from the contractile elements of a muscular cell: the fibrillary structures of a muscle fibre show a double refraction of form (Doppelformbrechung) along with double refraction proper; the fibrillae of the flagellum display the former type of refraction alone.

Hence it may be inferred that the double refraction proper is not absolutely necessary for a structure to be contractile, a definite orientation of mycellae serving to this purpose.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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MICROBIOLOGY

FORMATION OF NECROSES ON LEAVES OF A HYBRID *NICOTIANA TABACUM* × *N. GLUTINOSA*, INDUCED BY THE VIRUS OF TOBACCO MOSAIC UNDER CONDITIONS OF PLASMOLYSIS

By K. S. SUKHOV

(Communicated by A. A. Richter, Member of the Academy, 16.III.1944)

It is to the work done by Rischkov (1) that the investigation of phytopathogenic viruses owes one of its branches, *viz.* the study of the physiological conditions of virus accumulation in the vegetable organism. It is the physical aspect of those conditions that has attracted the attention of the present author.

The production of virus in a live cell seems, like any other biochemical work, to be intimately connected with the physical state of the protoplasm, in particular, with the phenomena of dehydration and peptization of its colloidal components. This note contains some data concerning the influence of plasmolysis upon the formation of necroses in plants affected by tobacco mosaic virus.

In our choice of the necrotic reactions we have been guided by certain data available in literature, according to which the accumulation of the virus is supposed to take place in the necrotic regions of the leaf tissue. Jensen (3) practised virus passages from separate necroses on leaves of *Nicotiana glutinosa*, and found the virus to preserve its normal activity after ten consecutive passages.

In order to obtain the necrotic reaction use was made of leaves of a hybrid *N. tabacum* × *N. glutinosa* derived by Prof. Ternovsky (2) who was kind enough to put some hybrid seed at our disposal.

As a rule, we had recourse to the method of leaf-halves, which had previously been applied by Holmes for the titration of the virus according to the number of necroses; the same method was also widely used by Rischkov when investigating the conditions of virus production in plants. It has recently been used also by Woods (4) on a similar occasion.

One of the leaf-halves was immersed to $^1/_2$ or $^2/_3$ of its length into sugar solution, the other was immersed to a similar depth into tap water. As vessels flat histological vials were used, with a capacity of 30—35 cm³. The experiment was run in the light at room temperature. Under these conditions necroses were found to appear by the second day, as a rule. In hypertonic sugar solutions the life-activity of plasmolysed tissues was evaluated after the flows of protoplasm observed in the epidermal cells. The leaves were infected with the virus by slightly rubbing their surface with a rubber-stopper moistened with juice pressed out of leaves of a diseased tobacco plant. Immediately upon infection, the leaf was washed with tap water and cut longitudinally into two parts which served for the experiment.

In order to produce plasmolysis, hypertonic solutions of mannose, maltose, saccharose and xylose were taken. The first observations on the half-leaves immersed were started after 24 hours. Within this time the tissues were found to have become strongly plasmolysed. The part immersed lost all of its turgor and, when taken from the solution, drooped as if it were dead. After two days, however, revived flows of protoplasm could again be observed in the epidermal cells of the immersed region of the leaf. By the second or the third day, necroses were found

to appear both on the experimental leaf-halves and on the controls. On the immersed areas of the experimental leaf-halves they were several times less in number than the corresponding areas of the control. At the same time their size in the plasmolysed area considerably exceeded that of the necroses of the controls (see table). The latter, as well as the necroses observed over the non-immersed area of the experimental halves, were small, almost point-like, whereas those of the plasmolysed part developed into large, round patches with a clear-cut concentrical structure. In individual experiments their size was 4, 5, 6 times that of the controls.

Plasmolysing substances	Concentration of solution, %	Number of necroses				Ratio of size of necroses in immersed parts in experiment and control
		Experiment		Control		
		immersed	unimmersed	immersed	unimmersed	
Mannose	20	44	43	126	212	8
»	20	16	no records	112	no records	5
»	20	8	» »	36	» »	5
Saccharose	20	11	26	28	33	6
»	20	3	14	23	19	6
Maltose	25	10	21	22	34	4
Xylose	20	0	14	20	8	5
»	20	0	12	11	11	5

Similar results have also been obtained with mannose, maltose and saccharose solutions. In the case of xylose, no necroses were found to develop on the immersed parts of the experimental leaf-halves; in the control they were also few, however. Yet those of them which have developed within the plasmolysed part of the leaf, near the line to which it was immersed into the solution, proved again five times the size of those in the control.

The experiment here described permit of the conclusion that the effect of intense plasmolysis is to reduce the number of necroses, on the one hand, and to make the necroses rapidly to extend to a larger area, on the other. The decrease in the number of necroses may possibly be due to a partial dying-off injured leaves, that served as a gate for the virus infection. As to the increase in the area of the necroses, we feel inclined to associate it with the gradual accumulation of the virus and its diffusion in the tissues. Consequently, the virus should be supposed to accumulate more rapidly in a plasmolysed tissue than in a normal one.

A comparison of the titre of virus taken from normal necroses and from necroses stimulated to growth by plasmolysis, would probably help to put this statement upon a solid foundation.

On the basis of the experiments carried out by the method here described it is impossible altogether to exclude the possibility of a biochemical influence of the part of the sugars introduced upon the trend of the fermentation processes associated with the carbohydrate metabolism of the cell, which might also tell upon the rate of virus accumulation. However, the fact that similar results have been obtained with solutions of mono- or disaccharoses, of normal or inverted sugars, make us suppose that in all these cases a similar mechanism of reaction is operative, and associate it with the phenomenon of plasmolysis and of dehydration of protoplasm by which it is attended.

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1945. Volume XLVI, № 1

PLANT PHYSIOLOGY

ON THE WORKING CONDITIONS OF THE FOLIAGE AS A FACTOR OF DIFFERENTIATION OF LATICIFEROUS VESSELS IN KOK-SAGHYZ ROOTS

By A. A. NICHIPOROVICH and E. F. IVANITZKAYA

(Communicated by A. N. Bach, Member of the Academy, 18.VI.1944)

We have shown in a previous paper (1) that the rhythm of formation of rings of laticiferous vessels in the roots of kok-saghyz and krym-saghyz appears to be constant and does not show any direct relation to the cycles of leaf formation. It remained to find out, however, whether the conditions of leaf formation and activity influence the intensity of differentiation of meristematic cells within individual rings, and whether they condition the abundance of laticiferous vessels in a given ring, their diameters and their rubber content.

In order to clear these points we have made the following experiment. Kok-saghyz was sown (on June 15th) on a well-fertilized and levelled field; on the 24th of July the area was subdivided into 4 equal plots and the plants growing on them were treated as follows:

1. The first plot served as a control.

2. Plants grown on the second plot were shaded with 3 layers of cheese-cloth in order to create conditions of highly reduced light.

3. On the third plot the bottom leaves were periodically removed from all the plants as soon as they reached full development; thus, only the upper young leaves were left and they continued to form until the end of the growing season. It was suggested that the specific physiological properties of the young leaves in which synthetic processes dominate over those of hydrolysis (2, 3) and where the processes of protein synthesis are especially pronounced, could affect the differentiation of the laticiferous vessels.

4. In the plants of the fourth group young leaves at the centre of the rosette were periodically removed, so that during the whole growing period the plants had to depend on the old leaves formed in the early half of the season.

All the experimental plants were harvested on September 28, for subsequent analysis. The general developmental features of the plants by this time were the following:

Table 1

Indices	Experimental Sets			
	1 (control)	2 (shading)	3 (removal of the lowest leaves)	4 (removal of the upper leaves)
Mean diameter of the root, mm	8.8	5.4	7.95	7.5
» weight of a fresh root, g	16.5	5.20	11.40	11.95
» » dry root, g	4.23	1.17	3.07	3.12
Per cent of rubber to dry root weight	7.62	4.81	7.99	8.37

The results of the anatomical examination of the roots are presented in Table 2.

Table 2

Sets	A Thickness of bark in mm	B Mean number of laticiferous vessel rings per root	C Mean distance between laticiferous vessels rings in mm (A:B)	D Mean number of parenchymatic cells between successive rings of laticiferous vessels	E Number of laticiferous vessels in 33° sector of best developed ring	F Mean diameter of fully developed laticiferous vessels in μ
1	3.70±0.16	11.15±0.24	0.333	10.0	40.43±2.19	24.1±1.14
2	2.34±0.11	7.87±0.53	0.292	10.0	28.41±1.93	23.7±1.12
3	3.38±0.10	10.64±0.45	0.315	10.2	49.24±2.17	30.3±0.66
4	3.41±0.14	10.61±0.40	0.326	10.0	31.30±1.87	31.7±0.49

The inspection of the data contained in Table 2 justifies the following conclusions as to the process of formation of the laticiferous vessels in relation to the working conditions and physiological condition of the foliage:

1. The rhythm of differentiation of the rings of laticiferous vessels in the roots of kok-saghyz is uniform and does not depend upon the conditions of growth of the roots. This is proved by the constancy of the number of cells between neighbouring rings of laticiferous vessels in the roots of all the experimental plants (column D).

2. The tempo of formation of the rings of the laticiferous vessels varies with the tempo of the growth of the plants themselves: the faster the growth of the root diameter, the faster the development of the plant, the more rings of the laticiferous vessels are formed. However, as the rhythm of the formation of rings remains constant, the numbers of rings ultimately produced by the plants are proportional to the thickness of the bark, the number of interstitial cells between the neighbouring rings being practically the same in all cases. The mean distance between the neighbouring rings of the laticiferous vessels in plants at identical developmental stages may vary within relatively narrow limits and chiefly owing to differences in the size of the intercalary parenchymatic cells; variation in their number is of a far lesser importance. The best developmental conditions result in greater parenchymatic cells and, consequently, in greater distances between the neighbouring rings of the laticiferous vessels (Table 2, column C).

3. While the rhythm of formation of rings of laticiferous vessels represents a character of considerable stability and independence of external influences, the intensity of formation of laticiferous vessels within an individual ring is a variable character which appears to be greatly influenced by the developmental conditions and by the function of the foliage (Table 2, column E). In the most general form the relation is borne out by the fact that the intensity of the formation of laticiferous vessels within a ring is the less, the worse are the working conditions of the foliage (set 2). This relation is actually far a more complex one, which is proved by the figures relating to sets 3 and 4. A preliminary conclusion may be drawn from them that a greater intensity of differentiation of the laticiferous vessels is stimulated by the presence of the young active leaves which are placed under favourable conditions of air nutrition (set 3 as compared to sets 4 and 1).

4. It has been shown by a number of studies ([4], [5]) that the total rubber content is closely linked with the diameter of the laticiferous vessels, the latter increasing with the age of the plant. The diameters of the laticiferous vessels are the greater, the better the conditions of nutrition. This simple interrelation, however,

is obscured by a number of other influences. In our experiment greatest diameters were found in plants which were deprived of leaves, irrespective of the kind of the latter, *i. e.* the effect was the same whether young or mature leaves had been removed. Possibly a definite rôle was played here by the reaction of the plant to the traumatic intervention (removal of the leaves), followed by repeated efflux of latex from the vessels.

The most variable characters and at the same time the most susceptible to growing conditions are, thus, the intensity of differentiation of the laticiferous vessels within individual rings and the increase in the diameter of the vessels. Treatments intended to increase the rubber content of the raw material should in the first line produce most favourable changes in these very indices.

Examination of the mass material suggests that the diameter of the laticiferous vessels and their abundance within the rings is highly variable both phenotypically and genotypically. This opens exceedingly favourable opportunities both for breeding new varieties of higher rubber content and for designing most suitable conditions of cultivation.

In concluding an additional factor which conditions the rubber content of the root should be evaluated, namely the degree to which the laticiferous vessels are filled with rubber. We were unable to determine it directly and to express it in absolute coefficients (*e. g.*, in gms of rubber per 1 cm^3 of volume of the vessels). Instead, we made an attempt to give a relative expression in taking ressort to the following calculation: from columns C, E and F, Table 2, we have derived the relative value of the «structural factor of rubber content», the control taken for unity. As an example may serve the comparison between roots of the 1st and 2nd sets. Let us take the relative development of the laticiferous vessels in the control for 1; the roots of set 2 should then be as many times reacher in rubber content than those of set 1, as many times smaller are the distances between the rings of their laticiferous vessels (provided in both cases other conditions are similar and the vessels are equally filled with rubber). In the case in question the index will be $0.333/0.292 = 1.14$. However, roots of set 2 contain fewer vessels per ring (column F), so that, *ceteris paribus*, their relative rubber content should be equal to $28.4/40.4 = 0.7$. The mean diameter of the laticiferous vessel is also less favourable in these roots. As rubber content depends upon the relative volumes of the laticiferous vessels, the rubber content of individual roots will obviously be under identical conditions proportional to the ratio of squares of the diameters of the laticiferous vessels; in our case it will be $23.7^2/24.1^2 = 0.97$. By multiplying the three indices we shall obtain the ultimate index of the relative «structural factor of rubber content». In the instance discussed it is $1.14 \times 0.70 \times 0.97 = 0.77$; it should be taken to mean that the ratio of the volume of the laticiferous vessels to the volume of the roots of the plants which were subjected to shading is 0.77 of the corresponding figure in the control. The vessels being equally filled with rubber in both instances, the same ratio should exist between rubber content of these roots.

As a matter of fact, however, the relative rubber content does not conform to expectation; as shown by Table 1, it equals $4.81/7.62 = 0.64$. As differences in specific weight of roots in both sets change but relatively slightly, the above differences should be attributed mainly to an unequal filling of the laticiferous vessels with rubber. Taking this index to be equal to 1 for the control, in the case of the 2nd set it will be $0.64/0.77 = 0.83$.

A similar treatment of the materials relating to other sets gave the following results (Table 3).

The following conclusions may be drawn from the data of Table 3. Light- and air-starvation impair both the developmental conditions of the laticiferous vessels and the degree of the filling with rubber, which ultimately results in a strong reduction in rubber content (variant 2).

Traumatic intervention in the form of removal of leaves stimulates noticeably the development of the laticiferous vessels; when young leaves are predominately left on the plant, a considerable increase in the number of vessels per ring takes place.

Table 3

Sets	Index ratio, the control taken for unity			Relative expression of the «structural factor of rubber content»	Rubber content		Relative degree of filling of the laticiferous vessels with rubber
	Distance between the rings of laticiferous vessels	Number of laticiferous vessels per ring	Diameter of the laticiferous vessels ($d_1^2 : d_2^2$)		in per cent to dry mass of the root	in relative indexes	
1	1.00	1.00	1.00	1.00	7.62	1.00	1.00
2	1.14	0.70	0.95	0.76	4.80	0.63	0.83
3	1.06	1.22	1.55	1.92	7.99	1.06	0.55
4	1.02	0.77	1.70	1.33	8.37	1.10	0.83

A considerable decrease in the degree of filling with rubber occurs in this case, however. Variation in this index dependent on the age was found by us in our previous works ([2], [5]). It was evident that not only the age as such is important here, but the degree of maturity of the leaves, too. As may be seen from Table 3, the presence of mature leaves ensures a better filling of the laticiferous vessels with rubber; on the other hand, preservation of young leaves (under normal conditions of light- and carbon-nutrition) results in a more intense formation of laticiferous vessels which are, however, less filled with rubber (sets 3 and 4).

It is of interest that the age of leaves determines the tempo of the stage development of the plants: while the proportion of blooming individuals among the control plants (set 1) was equal to 62 per cent, it attained 44 per cent in the set 2, and only 31 in the set 3; besides, the plants of the sets 2 and 3 bloomed considerably later.

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PLANT PHYSIOLOGY

PHOTOSYNTHESIS AND DEVELOPMENT OF PLANTS

By A. A. RICHTER, Member of the Academy, K. T. SUKHORUKOV and L. A. OSTAPENKO

We possess but a rather scanty knowledge as to the relation between photosynthesis and the development of a green plant. The leading position in this problem belongs to Klebs (1). From his own experiments with algae, carried on in the nineties of the last century, and with higher plants (1918) Klebs came to a conclusion that an outstanding part in the development of plants belongs to the assimilation of carbohydrates (2). In his papers Klebs often returned to photosynthesis, assimilation of CO_2, but he treated them essentially as conditions favourable for the accumulation of organic matter, and he never made any photosynthesis determination.

Klebs' ideas were more than once checked up and gave rise to prolonged discussions, the analysis of which is outside the scope of the present note. In the course of their investigation of photosynthesis and of its relation to the content of pigment in the cell, Willstätter and Stoll (1918) made many determinations of photosynthesis under constant optimal external conditions (3). They recorded considerable fluctuations in the content of chlorophyll, as well as different assimilation number, but the energy of the photosynthesis of the leaf was not found by them to be subject to any appreciable variation during the period of growth. The development of the plants under observation was going on, of course, but without affecting the photosynthesis ability of the leaf, if we are to judge by the figures reported by the authors cited above, without reference to the developmental stages of the plants.

Assimilation of CO_2 in spring and summer plants at different development stages was studied by M. Spaning (4). She succeeded to bear out with sufficient clarity that the plants are endowed with the lowest assimilation energy at the period of flowering. It is at this period that the highest transpiration is observable. In other words, in a plant high traspiration is accompanied with a low photosynthesis. Spaning supposes that the disintegration of proteins in the leaves and the reflux of disintegration products to the flowers also take place during this period. She made no determinations of the nitrogenic substances, neither did she describe the method by which photosynthesis determinations were made; but it is much likely to have been the gasometric method for plants grown under natural conditions. We cite here but a few statements, but they are sufficient to show that there is no unanimity in opinion on the subject; as to the problem of the relation between photosynthesis and the development of the plant, it is obviously at the initial stage of elaboration.

The purpose of the present work was to find out the dependence of photosynthesis on the development of the plant. Ten plants belonging to 5 families were under experiment; as to their development stage, they were either at the stage of flower ripeness, or flower unripeness. Flower ripeness as a development stage was evaluated, according to whether the plant was ready for budding, flowering or fruiting. The development stages could arbitrarily be curtailed or protracted by exposing the plants to photoperiodic effects, by varying the rations of nitrogenic fertilizers*, or by taking under experiment rapidly developing forms (beet).

* The plants were kindly put at our disposal by M. Ch. Čajlachjan from among his experiments, for which our hearty thanks are due to him.

Data referring to the determination of photosynthesis and respiration are summed up in Table 1.

Table 1

Date	Plant	Development stage	Factor	CO_2 in mg per leaf area in dm^2	
				photosynthesis	respiration
31 VIII	*Rudbeckia* sp.	Flowering	Long day	9.15	1.90
		No flowering	Short day	9.80	2.21
30.VIII	*Nicotiana tabacum*	Budding	1 nitrogen ration	8.11	1.58
		No budding	1/4 nitrogen ration	8.68	1.06
30.VIII	*Soja hispida*	Fruiting	Short day	8.00	1.52
		No budding	Long day	7.86	1.11
28.VIII	*Phaseolus multiflorus*	Flowering	Long day	7.90	1.17
		No flowering	Short day	7.27	1.46
29.VII	*Spinacia oleracea*	Flowering	Long day	8.89	1.94
		No flowering	Short day	8.37	2.10
29.VII	*Beta vulgaris esculenta*	Flowering	Spring variety	10.10	1.30
		No flowering		10.12	1.70
8.VIII	*Beta vulgaris esculenta*	Flowering	Spring variety	7.52	0.98
		No flowering		10.06	3.25

Along with such plants as differed in their development stages, under experiment were also plants at the same stage; in part of these the buds were regularly stripped off. In this way in plants at similar development stages we mechanically disturbed the correlation between the leaf and the flowering shoot. The results of determination of photosynthesis and respiration in such plants are summed up in Table 2.

Table 2

Date	Plant	Development stage	Treatment	CO_2 in mg per leaf area in dm^2	
				photosynthesis	respiration
29.VII	*Helianthus annuus*	Flowering		12.15	4.95
		No flowering	Decapitated	12.24	4.32
8.VIII	*Helianthus annuus*	Flowering		11.70	2.60
		No flowering	Decapitaded	10.62	3.69
29.VII	*Cucumis sativus*	Flowering		12.05	3.40
		No flowering	Buds stripped off	11.90	3.50
29.VII	*Solanum lycopersicum*	Flowering		8.64	1.80
		No flowering	Buds stripped off	9.45	2.53
2.IX	*Nicotiana rustica*	Flowering		9.64	2.00
		No flowering	Buds stripped off	9.00	2.72

From the figures given in the tables it may be seen that the initial stages of photosynthesis, the assimilation of CO_2 and the elimination of oxygen in a leaf are independent of the development stage of the plant. Neither is there any correlation between the leaf and the reproductive organs of the plant.

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PHYSIOLOGY

TO THE QUESTION OF DETERMINING ANTISULPHANILAMIDE FACTORS IN BLOOD

By A. L. JUDELES, I. J. POSTOVSKY and T. I. KAZANTZEVA

(Communicated by L. A. Orbéli, Member of the Academy, 4.VI.1944)

It has been demonstrated earlier (1) that the introduction of sulphanilamide compounds, such as sulphanilamide, sulphapyridine, sulphathiazol (streptocide, sulphidine, norsulphazol) into animal blood *in vitro* causes the disappearance of its mitogenetic radiation, and that the latter may be restituted in full by adding small quantities of *p*-aminobenzoic acid. Therapeutically ineffective preparations, as N^4-acetylsulphanilamide, 2-methylsulphanilamide, do not possess this inhibiting effect in regard to the mitogenetic radiation of blood; neither *p*-aminobenzoic acid, nor *o*- and *m*-aminobenzoic acids, both isomeric with *p*-aminobenzoic acid, are able to restore the blood radiation after it has disappeared under the influence of sulphanilamide or of its active derivatives.

We see thus that the inhibition of the mitogenetic radiation of blood by active sulphanilamide preparations and its restoration by *p*-aminobenzoic acid reproduces exactly the well-known and firmly established laws of the bacteriostatic action of sulphanilamide compounds and the antisulphanilamide (inhibitory) effect of *p*-aminobenzoic acid (2).

The question as to the inhibitory effect of various substances on sulphanilamide compounds is not only of theoretical interest, the mechanism of the action of sulphanilamide preparations being not sufficiently clear as yet, but, as it seems to us, it deserves also special attention on the side of the clinician who is frequently faced with the existence of «sulphamidoresistant» forms and cases of disease.

The question presented itself to us, whether these «sulphamidoresistant» forms and cases of disease might be explained by qualitative changes in the intermediary metabolism of proteins in the organism and the arising in connexion with it of antisulphanilamide factors, *i. e.* of inhibitors of the effect of sulphanilamide compounds.

This conjecture seemed to find confirmation in numerous observations reported of late in the literature on the question. Thus, Lockwood (3), Mellon (4) have shown that inhibitory effect is produced not only by *p*-aminobenzoic acid, but also by the intermediary product of protein disintegration, by peptone. Collin and McLeod (5), after confirming the inhibitory action of different peptones, have established a similar effect of the products of enzymatic hydrolysis of casein and of the tissues of various organs (muscle, spleen, pancreas). They also demonstrated the presence of inhibitors in a bound form in urine. Manwaring (6) has reported that many normal tissues of mammals, as well as sterile arthritic exudates, contain an active factor able to neutralize the therapeutic effect of sulphanilamide compounds (7–11).

It seemed expedient to use the inhibitory effect on the mitogenetic radiation of blood, produced by sulphanilamide preparations, and its restitution by the inhibitors of this effect, which we have described above, for the detection of antisulphanilamide substances. To test the applicability of our method for the detec-

tion of such inhibitors as may arise in the cases of disturbance of intermediary protein metabolism, we began our experiments, like Lockwood (3), by determining the effect produced on sulphanilamide compounds by the intermediary product of protein decomposition, by peptone. We found that the ability of mitogenetic radiation, lost by blood as a result of introducing into it of 0.5 mg of sulphanilamide per 1 ml of blood, is restored to its full intensity by the addition of 0.25 per cent peptone solution (0.025 mg of peptone per 1 ml of blood).

The intensity of radiation in control tests, in which blood was used without additions, equalized 40 per cent *. After the introduction of sulphanilamide this intensity was practically nil and after the introduction of both sulphanilamide and peptone it equalized 38 per cent (average of six separate tests). If follows that peptone inhibits the effect of sulphanilamide expressed in its ability to inhibit the mitogenetic radiation of blood. This finding is in complete accordance with Lockwood's results (3) concerning the inhibition of the bacteriostatic effect of sulphanilamide compounds by peptone.

The discovery of such an effect of an intermediary product of protein metabolism supplied us with grounds for studying the inhibitory significance of the conditions of the functional state of the organism by means of determining the comparative effect of sulphanilamide on the mitogenetic radiation of the blood of normal subjects, on the one hand, and of persons suffering from alimentary distrophy, or from traumatic cachexia due to wounds, on the other. It cannot be doubted that in such cases a most important part is played by changes in general and intermediary metabolism.

According to the part played in its development by ectogenic, resp. endogenic factors, alimentary distrophy is characterized by greatly varying clinical symptoms. Notwithstanding this, as well as a certain relativity in the selection of the control group of sound subjects, the results of our investigations summarized in the table show a marked difference in the blood of these groups, as regards the effect of sulphanilamide on its mitogenetic radiation.

It can be seen in this table that sulphanilamide added to the blood of sound subjects regularly produced an inhibition of mitogenetic radiation in 100 per cent of the cases tested, as this was established by us earlier for the blood of sound laboratory animals. This inhibition, expressed as the average per cent of the decrease of the number of gemmulating yeast cells in comparison with the number of such cells after irradiation with blood without sulphanilamide, averaged 78 per cent (maximum 120 per cent, minimum 42 per cent).

At the same time out of 37 cases of alimentary distrophy investigated by us (all cases without any complication by other diseases) only in 5 this inhibition of mitogenetic radiation by sulphanilamide, characteristic of the blood of sound persons, was found. In 16 cases it was extremely weak or totally absent, *i. e.* the same as observed in experiments, in which such inhibitors as *p*-aminobenzoic acid or peptone were intentionally introduced. In the remaining 16 cases not only was there no inhibition of mitogenetic radiation, but even a phenomenon paradoxical for the action of sulphanilamide was observed, *i. e.* a stimulation of mitogenetic radiation. This latter observation deserves special attention.

The number of investigated cases of traumatic cachexia is as yet insufficient for drawing any conclusions. However, in 3 cases only out of a total of 12 the addition of sulphanilamide to the blood provoked an inhibition of its mitogenetic radiation, *i. e.* an effect similar to its action on the blood of sound subjects. In 6 cases the inhibition was practically absent, while in 3 an enhanced intensity of mitogenetic radiation, similar to that in the blood of subjects suffering from alimentary distrophy, was observed.

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* The intensity of mitogenetic radiation was determined according to the method of A. Gurvich, by comparing the number of gemmulating yeast cells subjected to the effect of the mitogenetic radiation of blood with that in the control without radiation (12). The increase in the cell number is given in per cent to their initial number and reflects the relative intensity of the radiation. Every test was done simultaneously with and without the inhibitor (peptone), the same blood being used for every individual test.

Mitogenetic Radiation of the Blood of Sound Subjects and of Persons Suffering from Alimentary Distrophy or Wound Cachexia in the Presence or Absence of Sulphanilamide

Object of investigation	Blood of sound subjects	Blood of persons suffering from alimentary distrophy	Blood of persons suffering from wound cachexia
Number of cases	13	37	12
Average increase in gemmulating yeast cells, in %			
Control: blood and distilled water	41	34	32
Test: blood and sulphanilamide (1:2000)	9	35	27
Inhibition of the mitogenetic radiation of blood by sulphanilamide (average of all investigated cases)			
	78	0	15

Summarizing all that has been said above, it seems possible to explain this peculiar property of the blood in the case of alimentary distrophy and traumatic cachexia, *i. e.* conditions, characterized by qualitative and quantitative changes in the intermediary metabolism, in particular of the protein one, by the presence of antisulphanilamide factors, *i. e.* of inhibitors of the effect of sulphanilamide compounds.

It is difficult as yet to say—until the question is studied on a more considerable number of cases—to what degree it is possible to predict the therapeutic effect of sulphanilamide compounds in individual cases or forms of disease. The results of our study give, however, grounds to draw the attention of the clinicians to such conditions of the organism's functional status as may have an inhibitory influence on the therapeutic effect of sulphanilamide compounds. Special studies should indicate, how to change the conditions leading to this inhibition, and thus to create foundations for a more effective therapeutic action of sulphanilamide compounds.

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